Mechanical Behaviour of Materials

Chapter 01

Stress and strain (Review)

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Definition of stress

$$\sigma_{ij} = \begin{vmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{vmatrix}$$

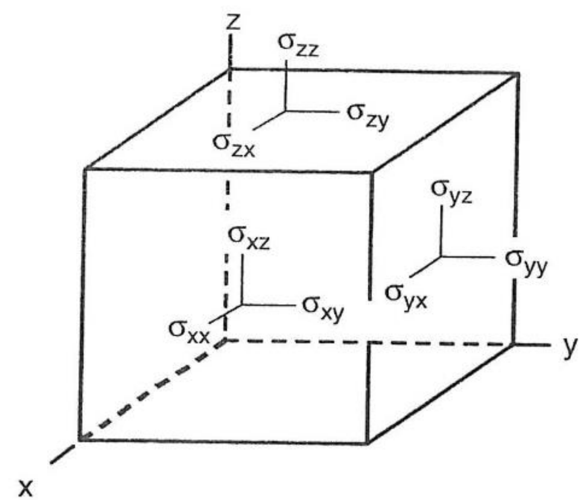
$$\sigma_x \equiv \sigma_{xx}$$

$$\tau_{xy} \equiv \sigma_{xy}$$

σ_{xy}

force direction

face normal

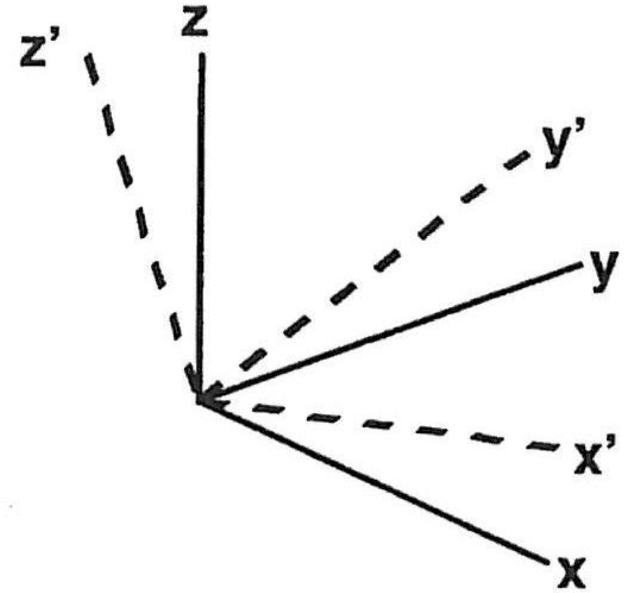


$$\sigma_{ij} = \frac{F_j}{A_i}$$

3D Stress Transformation

$$\sigma_{ij} = l_{im} l_{jn} \sigma_{mn}$$

$$\sigma_{ij} = \sum_{n=1}^{3} \sum_{m=1}^{3} l_{im} l_{jn} \sigma_{mn}$$



$$\sigma_{x'y'} = l_{x'x} l_{y'x} \sigma_{xx} + l_{x'x} l_{y'y} \sigma_{xy} + l_{x'x} l_{y'z} \sigma_{xz}$$
$$+ l_{x'y} l_{y'x} \sigma_{yx} + l_{x'y} l_{y'y} \sigma_{yy} + l_{x'y} l_{y'z} \sigma_{yz}$$
$$+ l_{x'z} l_{y'x} \sigma_{zx} + l_{x'z} l_{y'y} \sigma_{zy} + l_{x'z} l_{y'z} \sigma_{zz}$$

Stress Transformation

$$\sigma_{x'x'} = l_{x'x} l_{x'x} \sigma_{xx} + l_{x'x} l_{x'y} \sigma_{xy} + l_{x'x} l_{x'z} \sigma_{xz}$$
$$+ l_{x'y} l_{x'x} \sigma_{yx} + l_{x'y} l_{x'y} \sigma_{yy} + l_{x'y} l_{x'z} \sigma_{yz}$$
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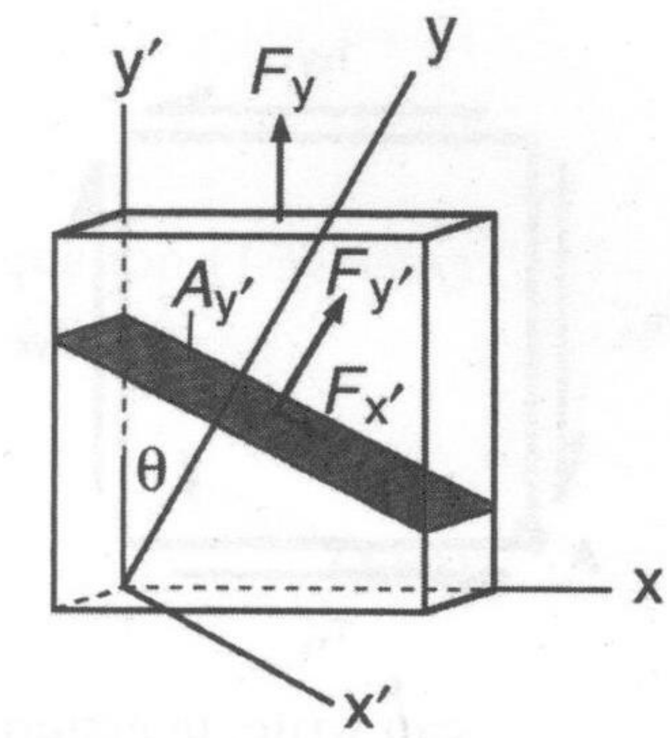
2D Stress transformation

$$\sigma_{y'} = \sigma_{y'y'} = F_{y'} / A_{y'} = \left(F_y \cos\theta\right) / \left(A_y / \cos\theta\right) = \sigma_y \cos^2\theta$$

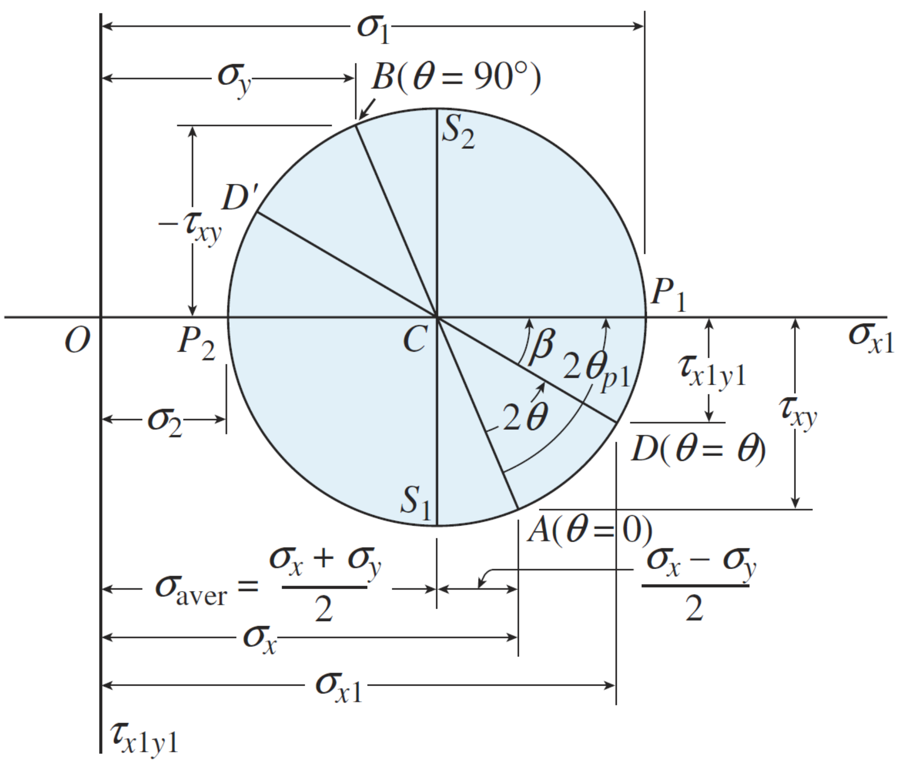
$$\tau_{y'x'} = \sigma_{y'x'} = F_{x'} / A_{y'} = \left(F_y \sin\theta\right) / \left(A_y / \cos\theta\right) = \sigma_y \cos\theta \sin\theta$$



$$\sigma_{y'} = l_{y'y}{}^2 \sigma_{yy} = \sigma_y \cos^2\theta$$

$$\tau_{x'y'} = l_{x'y} l_{y'y} \sigma_{yy} = \sigma_y \cos\theta \sin\theta$$

Mohr's Circle for Stress



- Mohr's Circle– The transformation equations plotted in graphical form.
- Rearranging the transform equations into equation of a circle in standard form:

$$(\sigma_{x1} - \sigma_{aver})^2 + \tau_{x1y1}{}^2 = R^2$$

- Mohr's Circle is typically plotted in two forms:

1) τ_{x1y1} Positive downward
2) τ_{x1y1} Positive upward

Definition of strain

Engineering strain

$$\varepsilon = \Delta l / l_0$$

True strain

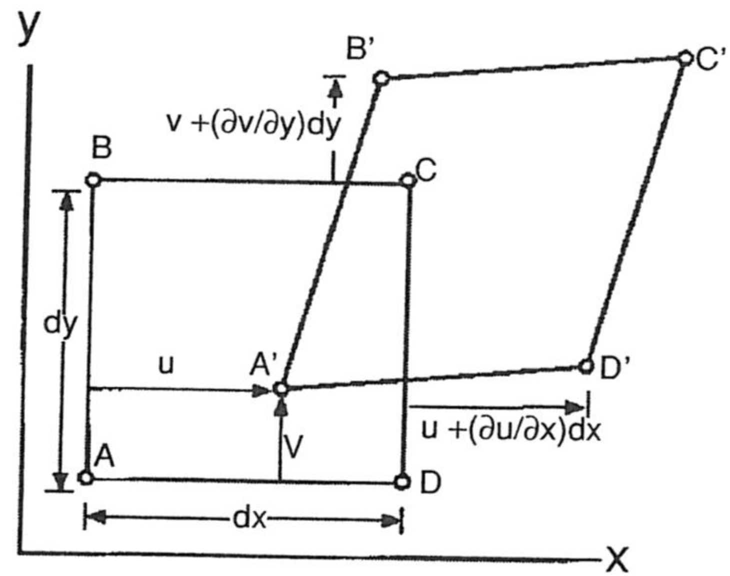
$$d\varepsilon = dl \big/ l_0$$

$$\varepsilon = \int dL/L = \ln\left(L/L_0\right)$$

Normal strain

$$\varepsilon_{xx} = \left(\partial u / \partial x\right) dx / dx = \partial u / \partial x$$

$$\varepsilon_{yy} = \left(\partial v / \partial y\right) dy / dy = \partial v / \partial y$$



Shear strain

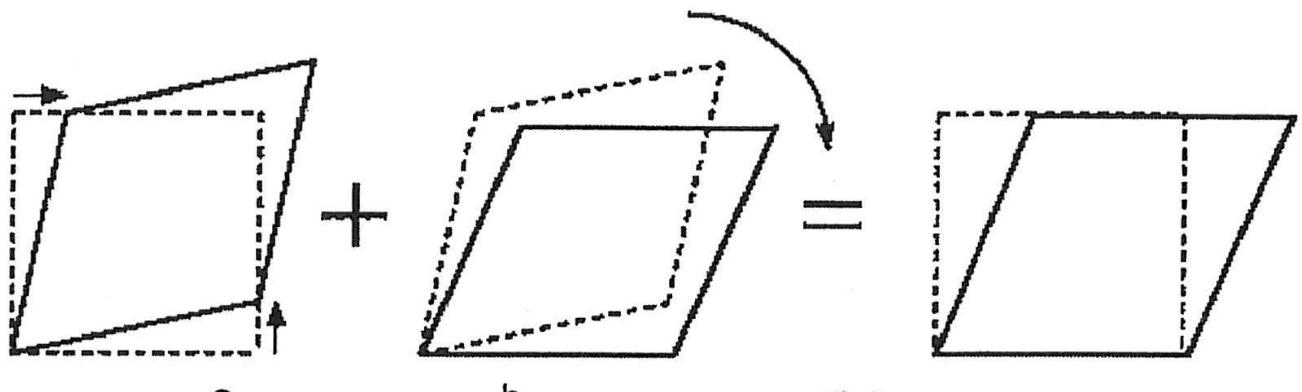
$$e_{yx} = \frac{\partial u_y}{\partial x} \qquad e_{xy} = \frac{\partial u_x}{\partial y}$$

$$\varepsilon_{ij} = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{yx} & \varepsilon_{zx} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{zy} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{vmatrix}$$

$$\varepsilon_{xy} = \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

$$\varepsilon_{yz} = \varepsilon_{zy} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)$$

$$\varepsilon_{zx} = \varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)$$



3D Strain Transformation

$$\varepsilon_{ij} = l_{im} l_{jn} \varepsilon_{mn'}$$

$$\varepsilon_{x'x'} = l_{x'x} l_{x'x} \varepsilon_{xx} + l_{x'x} l_{x'y} \varepsilon_{xy} + l_{x'x} l_{x'z} \varepsilon_{xz}$$
$$+ l_{x'y} l_{x'x} \varepsilon_{yx} + l_{x'y} l_{x'y} \varepsilon_{yy} + l_{x'y} l_{x'z} \varepsilon_{yz}$$
$$+ l_{x'z} l_{x'x} \varepsilon_{zx} + l_{x'z} l_{x'y} \varepsilon_{zy} + l_{x'z} l_{x'z} \varepsilon_{zz}$$

$$\varepsilon_{x'y'} = l_{x'x} l_{y'x} \varepsilon_{xx} + l_{x'x} l_{y'y} \varepsilon_{xy} + l_{x'x} l_{y'z} \varepsilon_{xz}$$
$$+ l_{x'y} l_{y'x} \varepsilon_{yx} + l_{x'y} l_{y'y} \varepsilon_{yy} + l_{x'y} l_{y'z} \varepsilon_{yz}$$
$$+ l_{x'z} l_{y'x} \varepsilon_{zx} + l_{x'z} l_{y'y} \varepsilon_{zy} + l_{x'z} l_{y'z} \varepsilon_{zz}$$

Mohr’s circle for strain