

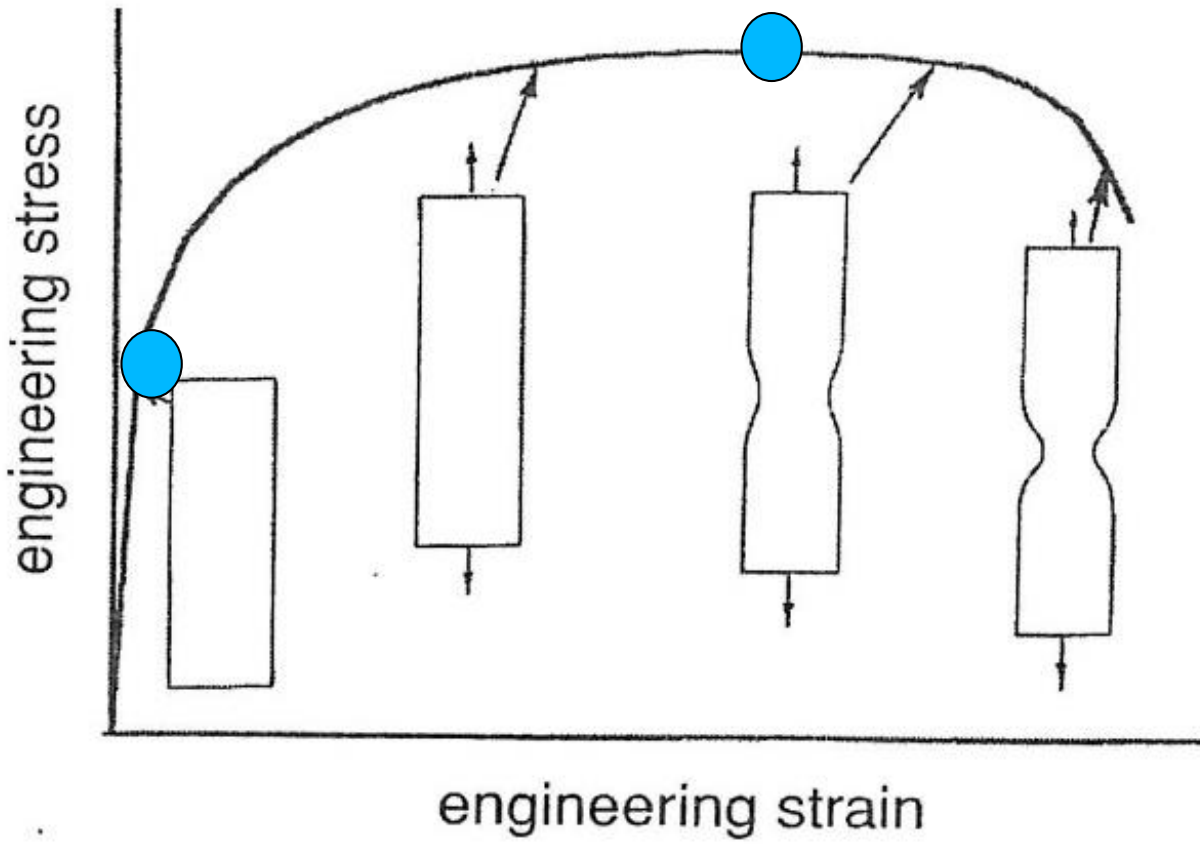
Mechanical Behaviour of Materials

Chapter 3

Mechanical testing: Hardness

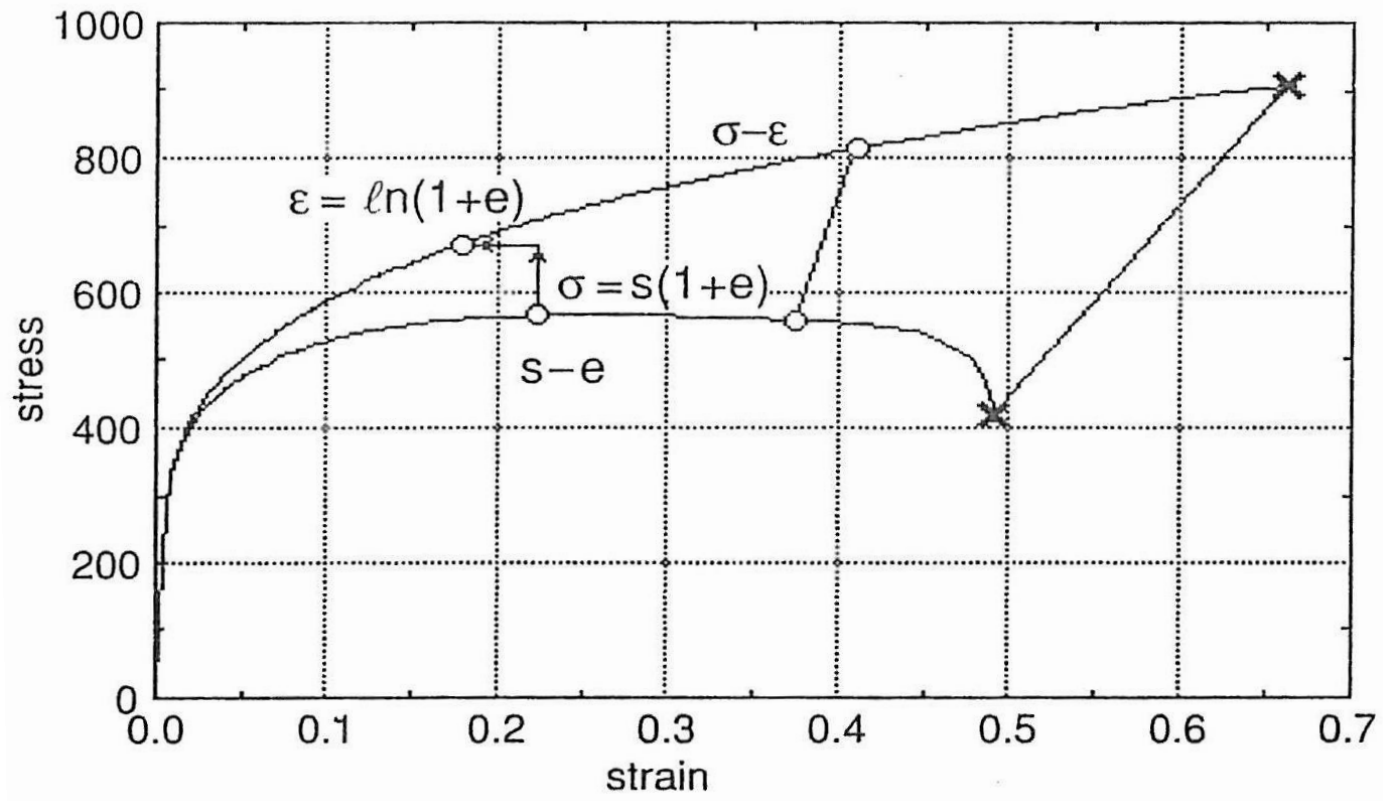
Dr.-Ing. 郭瑞昭

Stress-strain curve



True stress and strain

$$\sigma_t = \sigma(1 + \epsilon) \quad \epsilon_t = \ln(1 + \epsilon)$$



Compression test

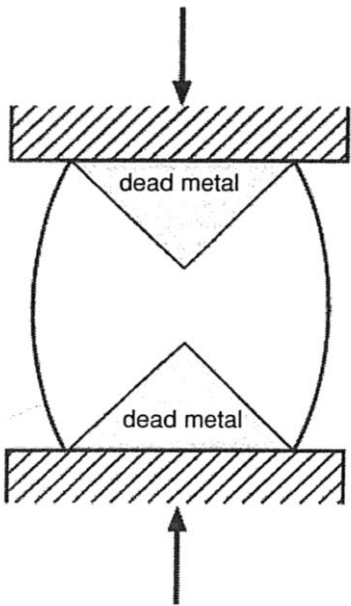
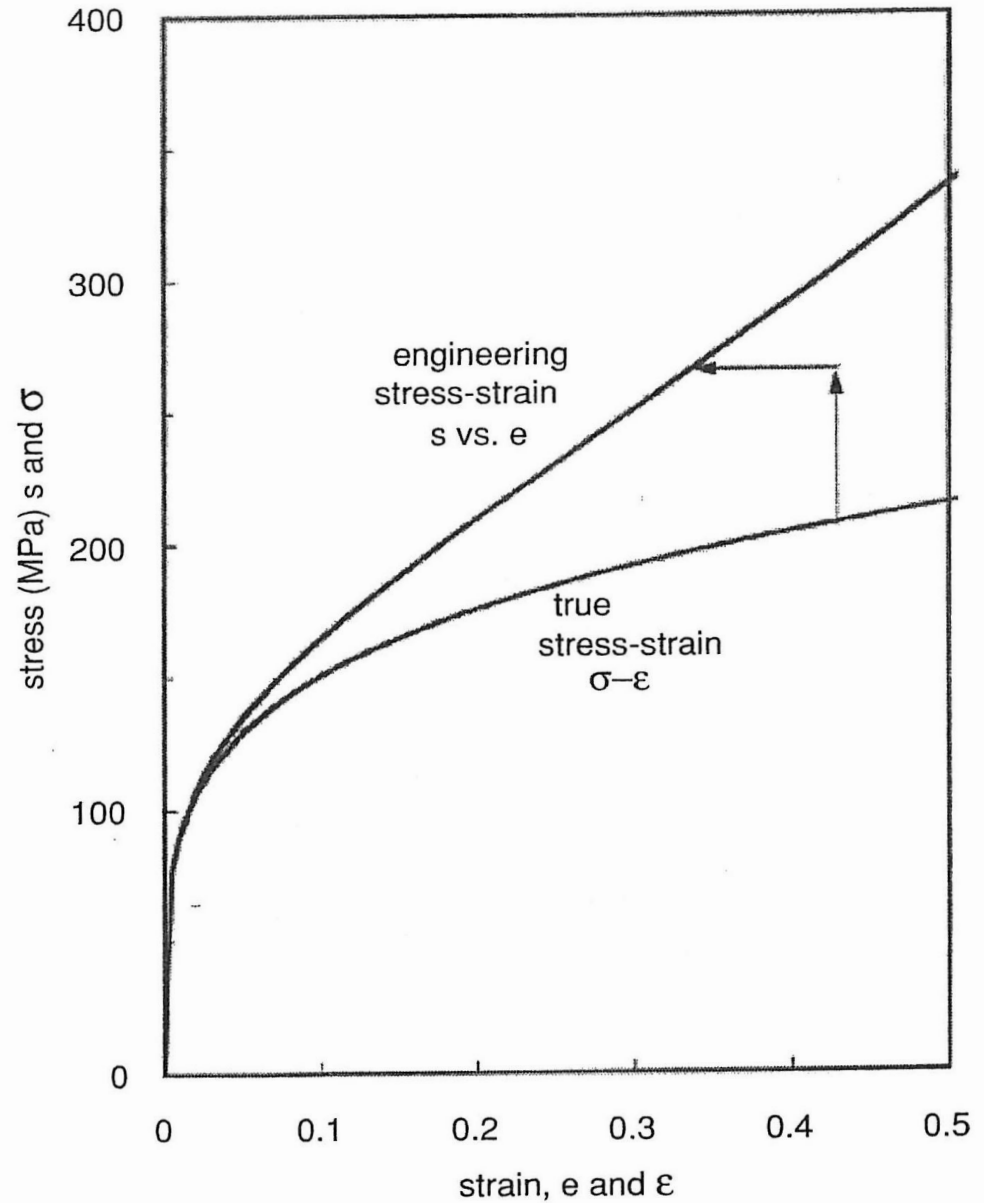


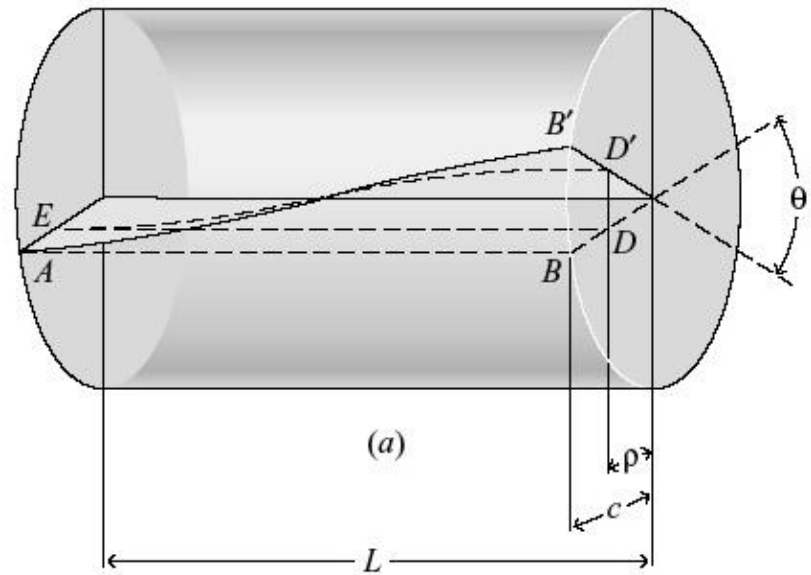
Figure 4.1. Unless the ends of a compression specimen are well lubricated, there will be a conical region of undeforming material metal at each end of the specimen. As a consequence the midsection will bulge out or *barrel*.



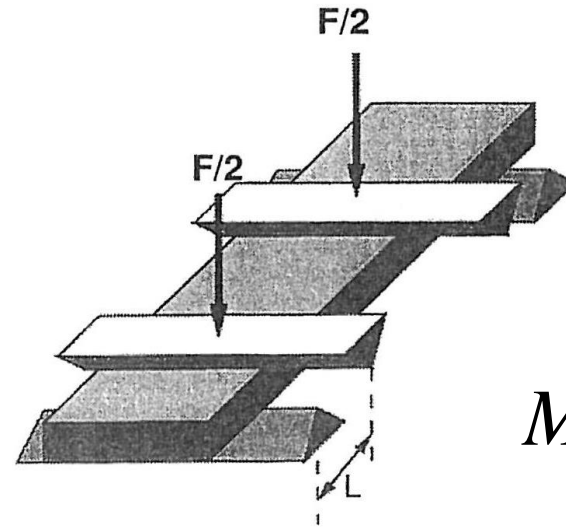
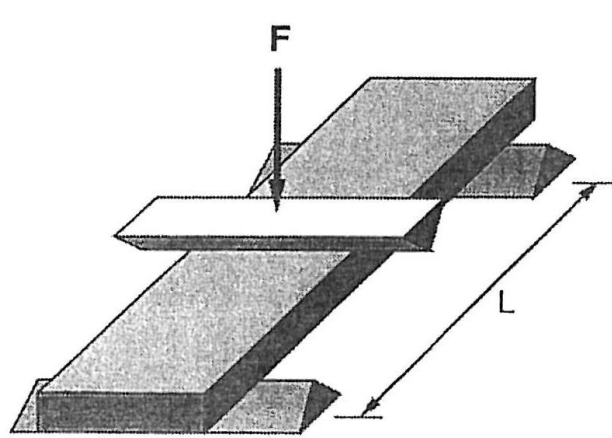
Torsion test

$$J_{\max} = \frac{\pi}{2} c^4$$

$$\tau_{\max} = \frac{T_{\max} \cdot c}{\frac{\pi}{2} c^4} = \frac{2T_{\max}}{\pi c^3}$$



Bending test



$$M_{\max} = \frac{FL}{4}$$

What is the moment in three point and four point bending?

$$\sigma_{\max} = \frac{M_{\max} c}{I} = \frac{M_{\max} \cdot h/2}{\frac{bh^3}{12}} = \frac{6M_{\max}}{bh^2} = \frac{3FL}{2bh^2}$$

Hardness tests

	Hardness	
	Vickers or Brinell	Mohs
	3000	10
Tantalum carbide	2000	Diamond
Alumina ceramics		9
	1000	8
Zircon ceramics		
Polycrystalline glass		7
Manganese steels	500	Quartz
	300	6
Titanium		Feldspar
	200	5
Carbon steels		Apatite
	100	4
		Fluorite
Aluminum		3
	50	Calcite
Magnesium		2
	30	Gypsum
	20	1
		Talc
	10	
Hard lead alloys		
	5	
Soft lead		
	3	
	2	

Indentation test are divided into three classes:

Nanoindentation

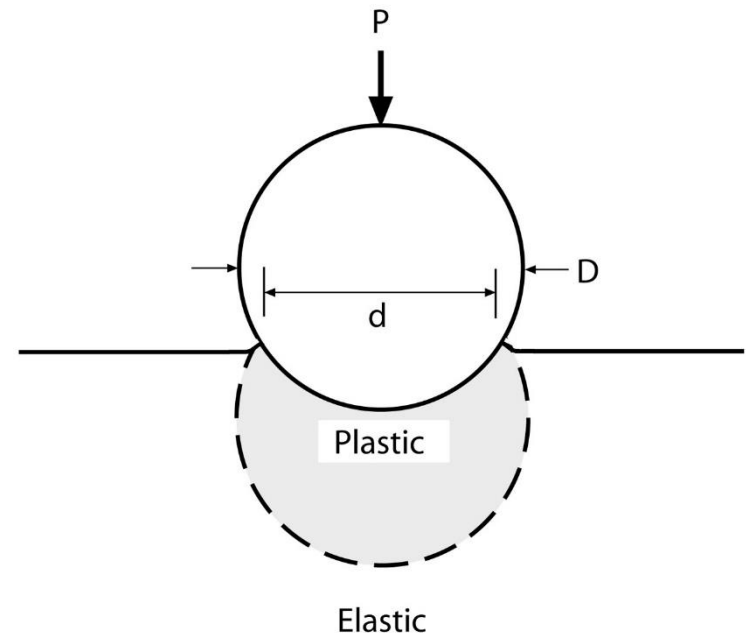
_____ mN

Microindentation

_____ 200 gf (about 2N)

Macroindentation

Hardness cannot be considered a fundamental property of a metal. It represents resistance to plastic deformation.



Hardness tests

Projected area:
Knoop,
nanoindentation

Contact area:
Brinell, Vickers,
Rockwell

Depth:
Rockwell

Shape of Indentation

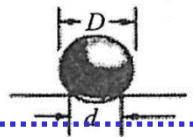
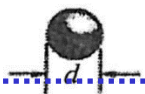
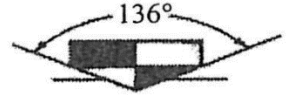

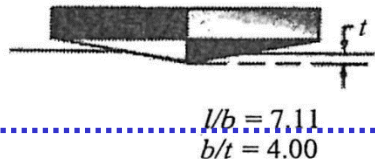
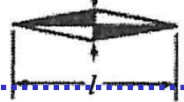
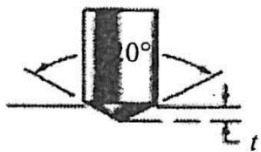

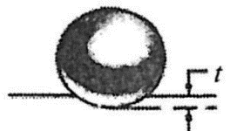



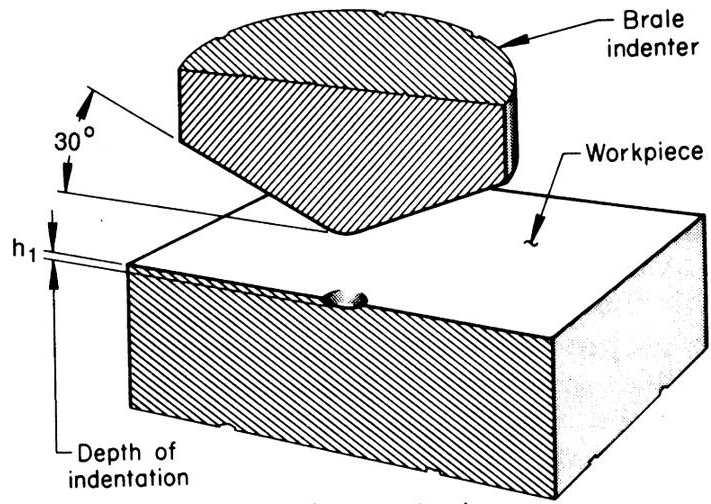
Test	Indenter	Side View	Top View	Load	Formula for Hardness Number
Brinell	10-mm sphere of steel or tungsten carbide			P	$BHN = \frac{2P}{\pi D(D - \sqrt{D^2 - d^2})}$
Vickers	Diamond pyramid			P	$VHN = 1.72 P/d_1^2$
Knoop microhardness	Diamond pyramid			P	$KHN = 14.2 P/l^2$
Rockwell					
A } C } D }	Diamond cone			60 kg 150 kg 100 kg	$R_A =$ $R_C =$ $R_D =$ } 100 - 500t
B } F } G }	$\frac{1}{16}$ in.-diameter steel sphere			100 kg 60 kg 150 kg	$R_B =$ $R_F =$ $R_G =$ } 130 - 500t
E	$\frac{1}{8}$ in.-diameter steel sphere			100 kg	$R_E =$

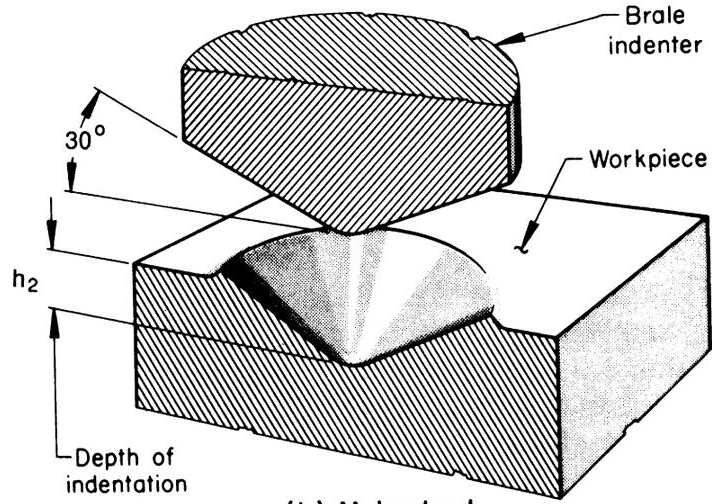
Figure 4.15. Various hardness tests. From H. W. Hayden, William G. Moffat, and John Wulff, *Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, Wiley, 1965.

Rockwell test

$$HBX = M - \frac{t}{0.002} = M - 500t$$



(a) Minor load



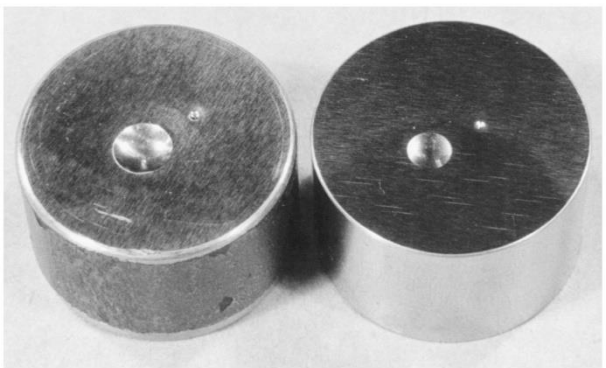
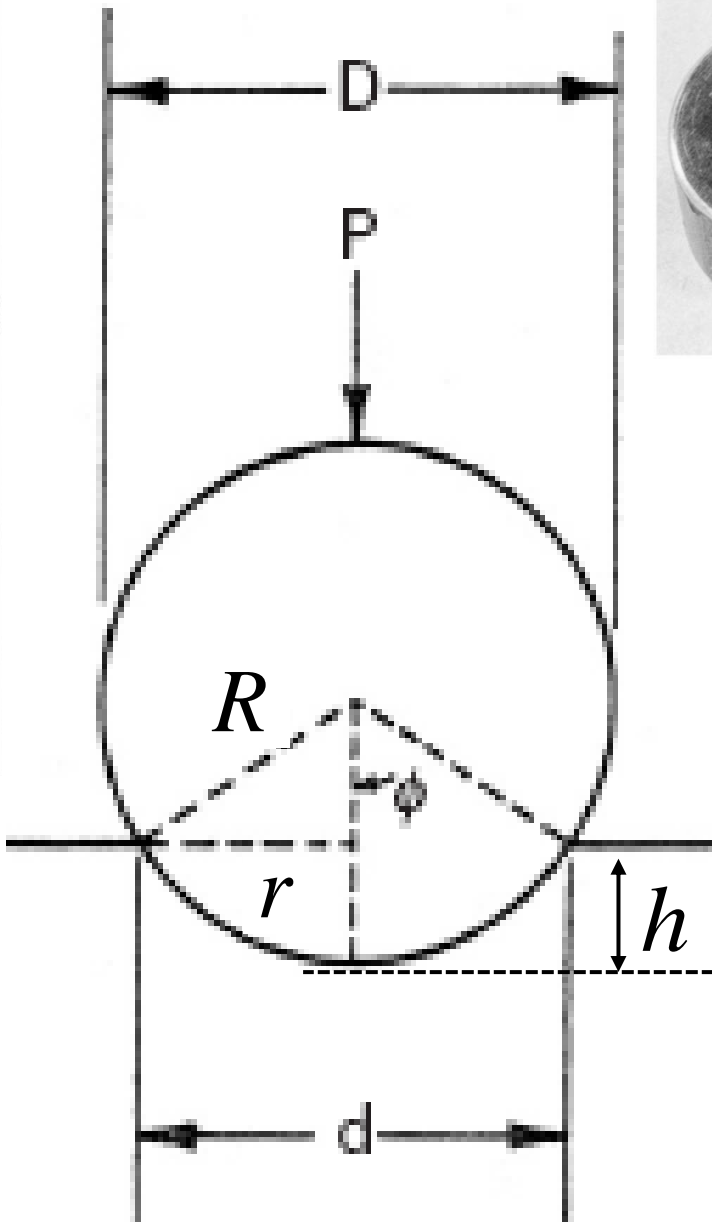
(b) Major load

Scales for Rockwell Hardness Tester

Table 3.3 | Details of the More Important Scales Available for the Rockwell Hardness Tester

Scale Designation	Type of Indenter	Major Load (kgf)	Typical Field of Application
A	Brale	60	The only continuous scale from annealed brass to cemented carbide, but is usually used for harder materials
B	1/16-in.-diameter steel ball	100	Medium-hardness range (e.g., annealed steels)
C	Brale	150	Hardened steel > HRB100
D	Brale	100	Case-hardened steels
E	1/8-in.-diameter steel ball	100	Al and Mg alloys
F	1/16-in.-diameter steel ball	60	Annealed Cu and brass
L	1/4-in.-diameter steel ball	60	Pb or plastics
N	N Brale	15, 30, or 45	Superficial Rockwell for thin samples or small impressions

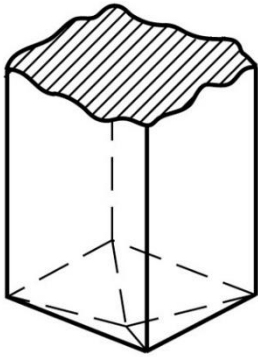
Brinell test (actual area)



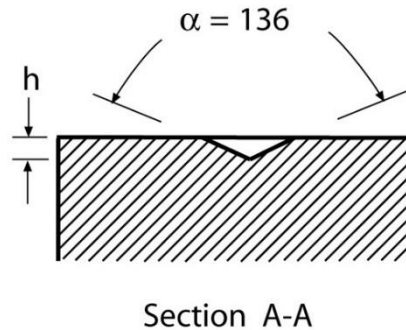
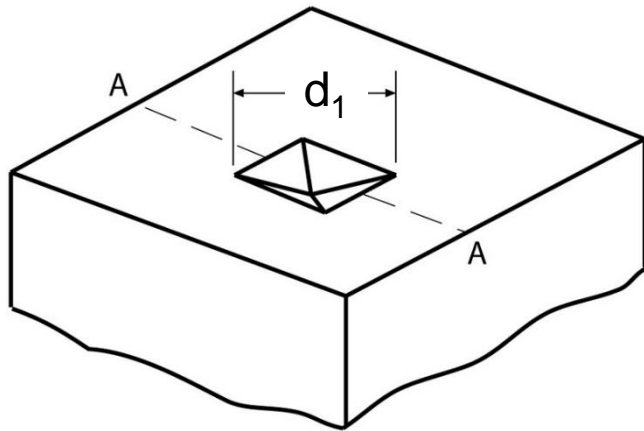
$$HB = \frac{P}{A_{actual}}$$

$$A_{actual} = 2\pi R \cdot h$$

Vickers Hardness Test (actual area)



$$HV = \frac{2P \sin(\theta/2)}{d_1^2} = \frac{1.85P}{d_1^2}$$



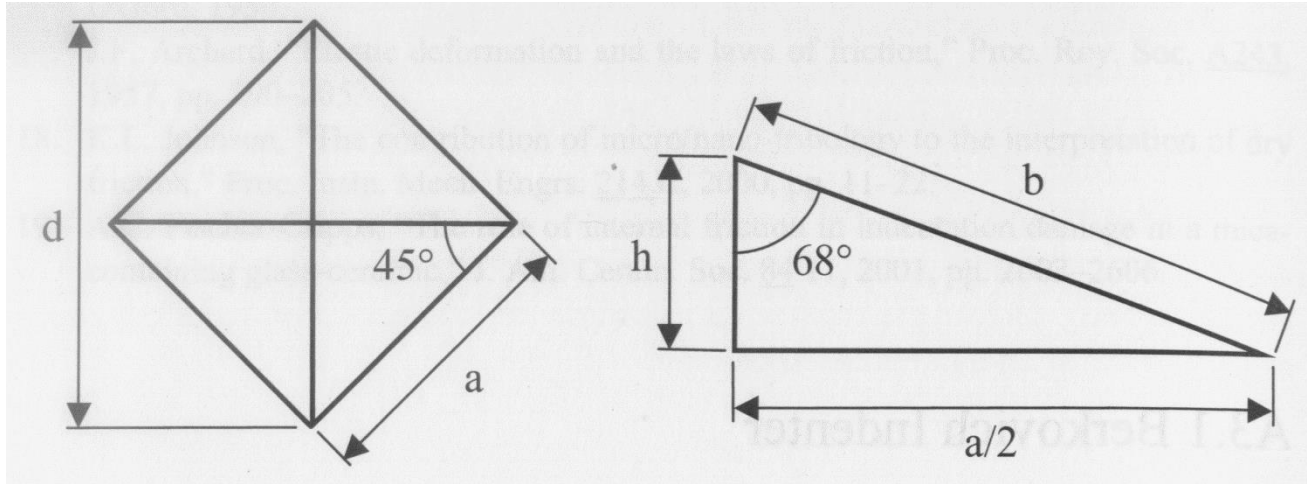
$$HV = \frac{P}{d^2} = \frac{1.85P}{d^2}$$

Relationships Between Yield Stress and Hardness

$$HV = 3\sigma_y$$

For non-work-hardening materials

Vickers indenter



Projected area

Surface area

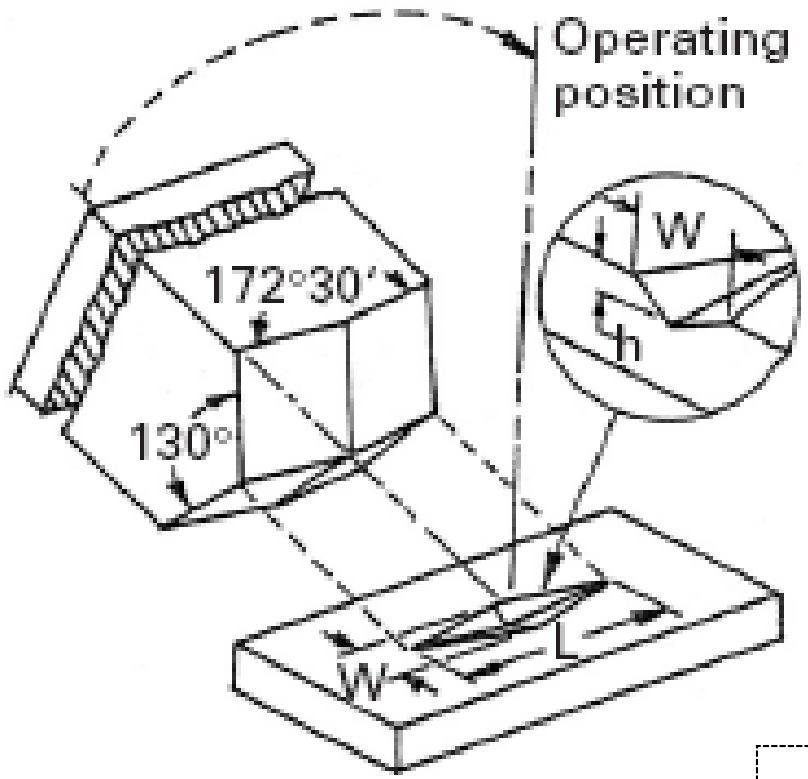
$$\sin 45^\circ = \frac{d/2}{a} = \frac{d}{2a}$$

$$A_{proj} = a^2 = \frac{d^2}{2}$$

$$\sin 68^\circ = \frac{a}{2b}$$

$$A_{surface} = 4 \frac{ab}{2} = \frac{a^2}{\sin 68} = \frac{d^2}{2 \sin 68}$$

Knoop Indenter (projected area)



Details of the Knoop indenter, together with its impression.

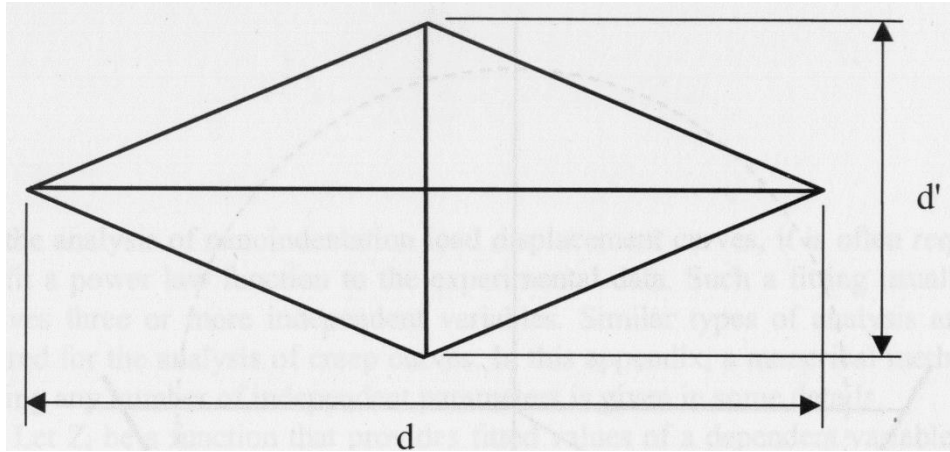
The load of kgf

$$KHN = \frac{14.2P}{d^2}$$

The length of the major diagonal, in mm

$$KHN = \frac{P}{0.070275d^2} = \frac{14.2P}{d^2}$$

Knoop Indenter



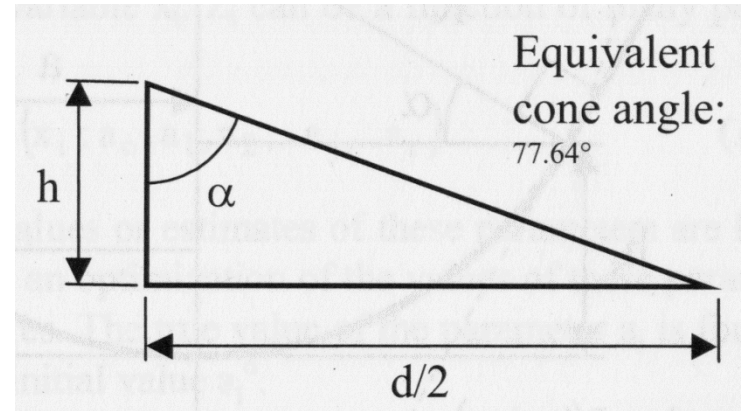
Projected area

$$A_{proj} = 2 \frac{d}{2} \frac{d'}{2} = \frac{d^2}{2} \cot \alpha_1 \tan \alpha_2$$

$$d = 2h \tan \alpha_1$$

$$d' = 2h \tan \alpha_2$$

$$d' = d \frac{\tan \alpha_2}{\tan \alpha_1}$$



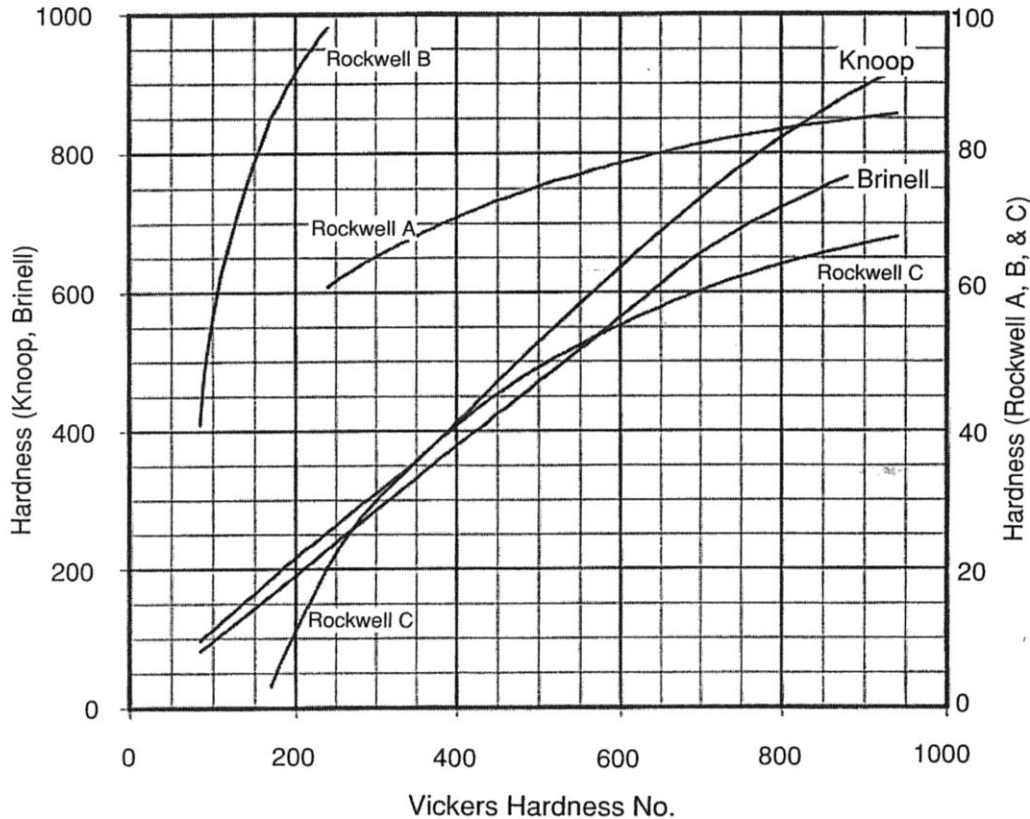
$$\alpha_1 = 86.25^\circ$$

$$\alpha_2 = 65^\circ$$

$$A_{proj} \approx 0.070275d^2$$

Yield strength

$$H = \left(2/\sqrt{3}\right) \left(1 + \pi/2\right) Y = 2.97Y \approx 3Y$$



$$B \approx 0.95V$$

$$K \approx 1.05V$$

$$R_C \approx 100 - 1480/\sqrt{B}$$

$$R_B \approx 134 - 6700/B$$

Figure 4.16. Approximate relations between several hardness scales. For Brinell (3000 kg) and Knoop read left. For Rockwell A, B, and C scales read right. Data from *Metals Handbook*, Vol. 8, ninth ed., American Society for Metals, 1985.

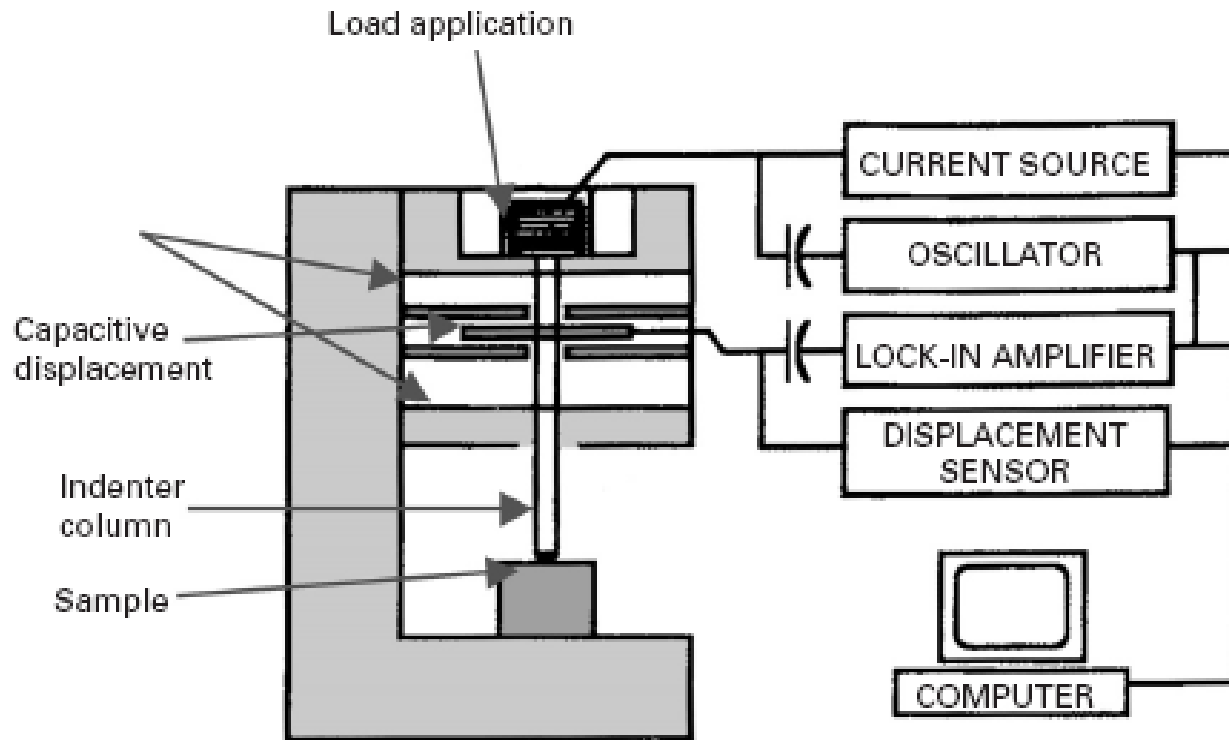
Mutual indentation hardness

The hardness as the indentation force/area, H , is proportional to its yield strength Y :

$$H = \left(2/\sqrt{3}\right)\left(1 + \pi/2\right)Y = 2.97Y \approx 3Y$$

$$H = cY$$

Nanoindenter



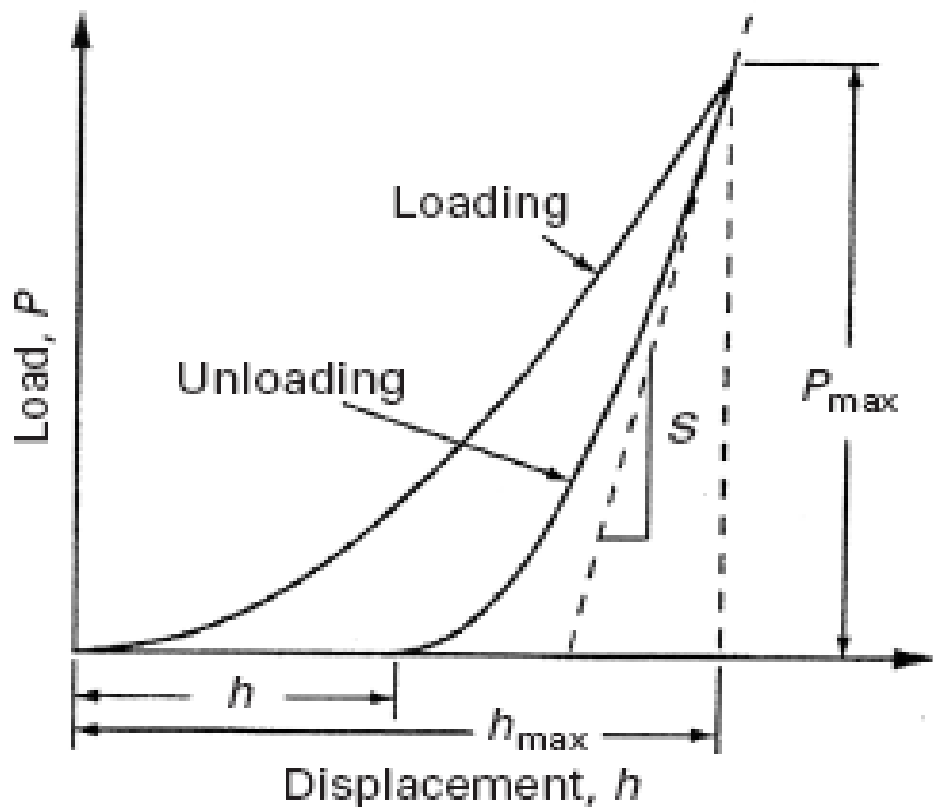
The instrument is a computer-controlled depth-sensing indentation system measuring extremely small forces and displacement.

Load vs. Indenter Displacement

$$E_r = (1 - \nu_i^2)E_i + (1 - \nu_s^2)E_s$$

$$H = \frac{P_{\max}}{A_{proj}} \quad \text{Projected area}$$

$$A = a + bh_i^{1/2} + ch_i + dh_i^{3/2} + 24.5h_i^2$$



$$S = \frac{dP}{dh} = \frac{2}{\sqrt{\pi}} E_r \sqrt{A} \quad \text{Stiffness}$$

$$E_r = \frac{\frac{dP}{dh}}{\frac{2}{\sqrt{\pi}} \sqrt{A}} \quad \text{Reduced modulus}$$