Mechanical Behaviour of Materials

Chapter 04-3

Fracture: Mechanisms

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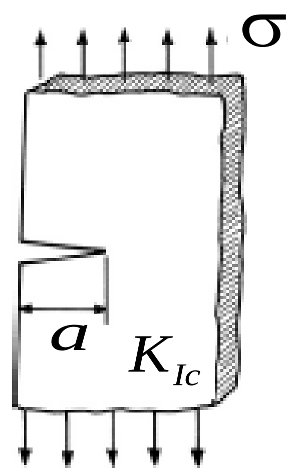
Influence Parameters of Facture Mechanics

A material fracture depends on temperature, the stress state and with time, and the environment.

Applied stress



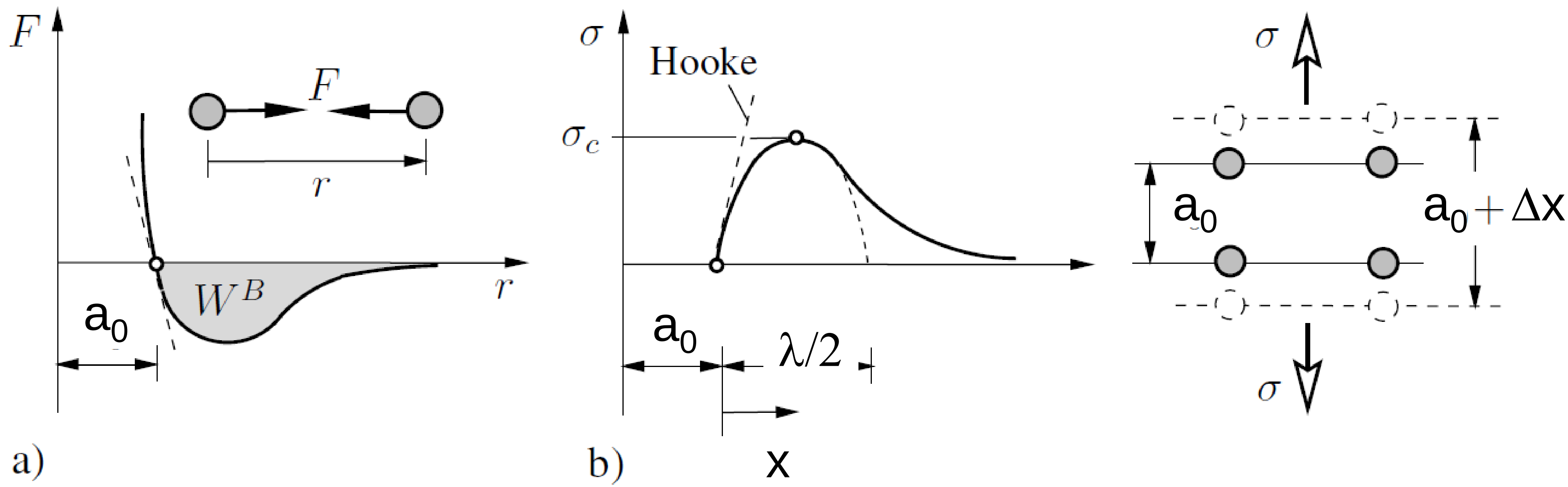
Flaw size

Fracture toughness

Theoretical fracture strength of brittle materials (stress approach)

Under normal stress a material is to cleave, when the fracture surface is perpendicular to the applied stress without cracks.

The atoms are separated along the direction of the applied stress.



$$F = F_{max} \sin\left(\frac{2\pi}{\lambda} x\right) \qquad \sigma = \sigma_{max} \sin\left(\frac{2\pi}{\lambda} x\right)$$

Derivation of theoretical fracture strength

For small displacement

$$\sigma = \sigma_{th} 2\pi \frac{x}{\lambda}$$

Following Hook's law

$$\sigma = \mathrm{E} \frac{x}{a_0}$$

$$\Rightarrow \quad \sigma_{th} = \frac{E\lambda}{2\pi a_0}$$

Theoretical facture strength (energy approach)

Fracture results in the creation of the two new surfaces, each surface energy γ_s per unit area.

The work of fracture W by the extended stress is expended in creating the new surfaces by breaking the atomic bonds.

$$W_{fracture} = \int_0^{\lambda/2} \sigma_{th} \sin\left(\frac{2\pi x}{\lambda}\right) dx = \sigma_{th} \frac{\lambda}{\pi} = 2\gamma$$

$$\sigma_{th} = \frac{\lambda}{2\pi a_0} E$$

$$\sigma_{th}^2 = \frac{\gamma \cdot E}{a_0} \qquad \sigma_{th} = \sqrt{\frac{E\gamma}{a_0}}$$

σ_{th}: maximum stress at the end of the major axis

σ_a: applied stress normal to the major axis

a: half major axis

b: half minor axis

List of theoretical fracture strength of some materials

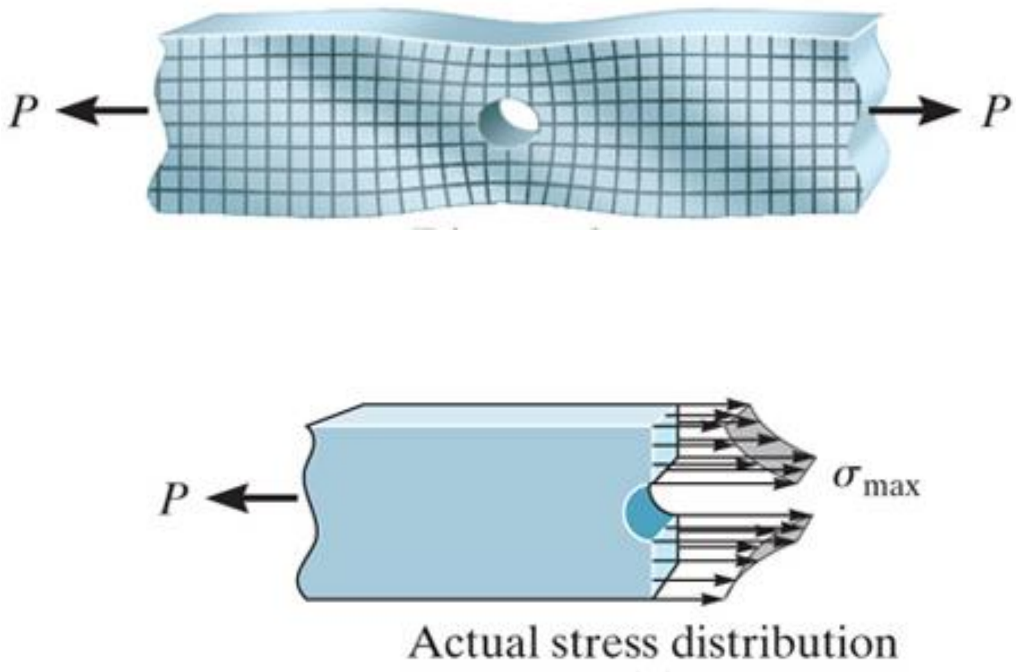
Table 7.1 Theorectical Cleavage Stresses According to Orowan's Theory*

Element	Direction	Young's Modulus (GPa)	Surface Energy (J/m2	σ_{max} (GPa)	σ_{max}/E
σ-Iron	<100>	132	2	30	0.23
	<111>	260	2	46	0.18
Silver	<111>	121	1.13	24	0.20
Gold	<111>	110	1.35	27	0.25
Copper	<111>	192	1.65	39	0.20
	<100>	67	1.65	25	0.38
Tungsten	<100>	390	3.00	86	0.22
Diamond	<111>	1,210	5.4	205	0.17

* Adapted with permission from A. Kelly, *Strong Solids*, 2nd ed. (Oxford, U.K.: Clarendon Press, 1973), p. 73.

Inglis Theory of Crack-initiated fracture (stress approach)

The fundamental requisite for the propagation of a crack is that the stress at the tip of the crack must exceed the theoretical cohesive strength of the material.

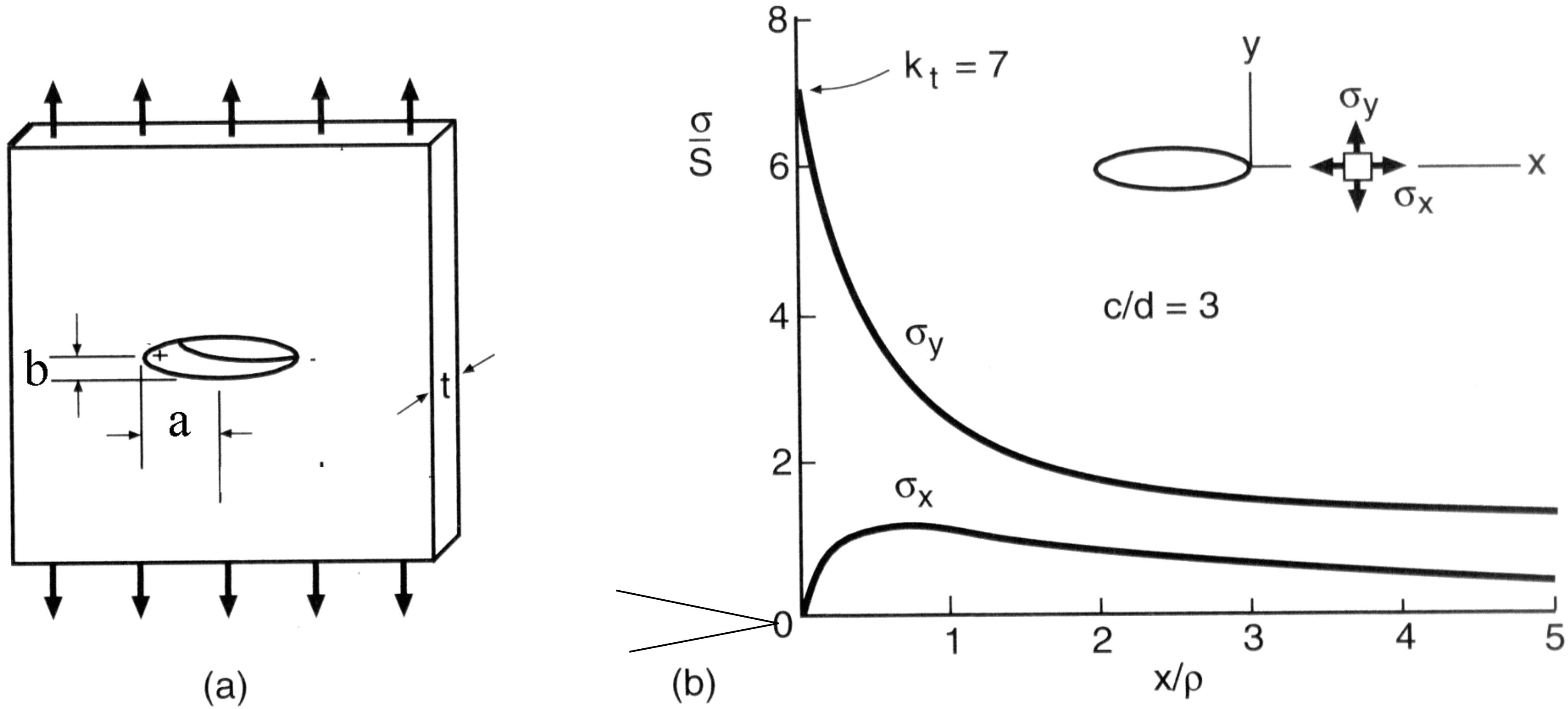


$$k_t = \frac{\sigma_{max}}{\sigma_{average}}$$

The stress concentration factor (SCF) is as the ratio of the maximum stress to the applied stress.

Stress concentration around an elliptic crack

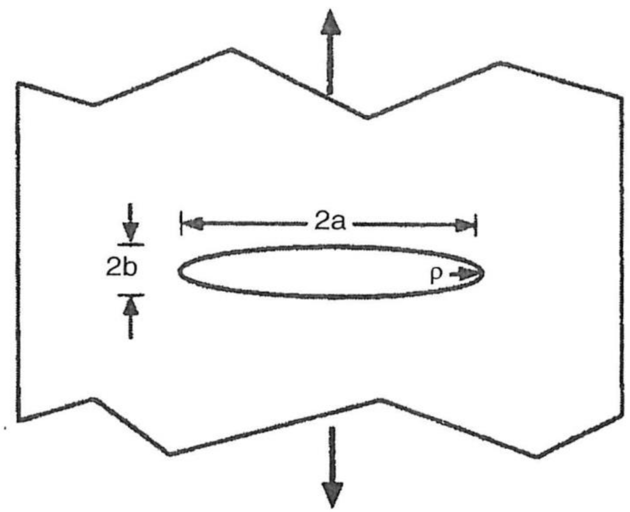
$$\sigma_{max} = \sigma_a \left(1 + 2\frac{a}{b} \right)$$



The stress distribution along the x-axis near an elliptic crack

Stress concentration factor k_t

Inglis: The stress rises dramatically near the hole and has a maximum value at the edge of the hole. The maximum value is given:



$$\sigma_{max} = \sigma_a \left(1 + 2\sqrt{\frac{a}{\rho}}\right)$$

$$\rho = \frac{b^2}{a}$$

the radius of curvature at the end of the ellipse

$$\frac{\sigma_{max}}{\sigma_a} \approx 2\sqrt{\frac{a}{\rho}}$$

$$a >> \rho$$

$$k_t = 2\sqrt{\frac{a}{\rho}}$$

stress concentration factor

$$k_t = \frac{\sigma_{max}}{\sigma_{aver}}$$

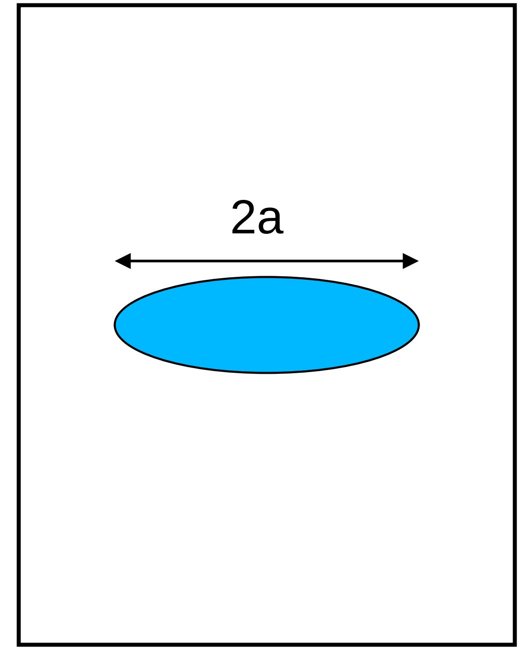
External applied stress ← $$\frac{\sigma_{max}}{\sigma_a} = 1 + 2\frac{a}{b}$$ → stress concentration factor

Griffith crack theory (Energy criterion γ_s)

thickness

$$U_{\text{surf}} = 2\,a \cdot t \cdot 2\gamma_s$$

$$U_{\text{el}} = \left(\frac{\sigma^2}{2E}\right)\left(2\pi a^2 t\right) = \frac{\pi\, a^2 t \sigma^2}{E}$$



$$\Delta U = 4at\gamma_s - \frac{\pi\, a^2 t \sigma^2}{E}$$

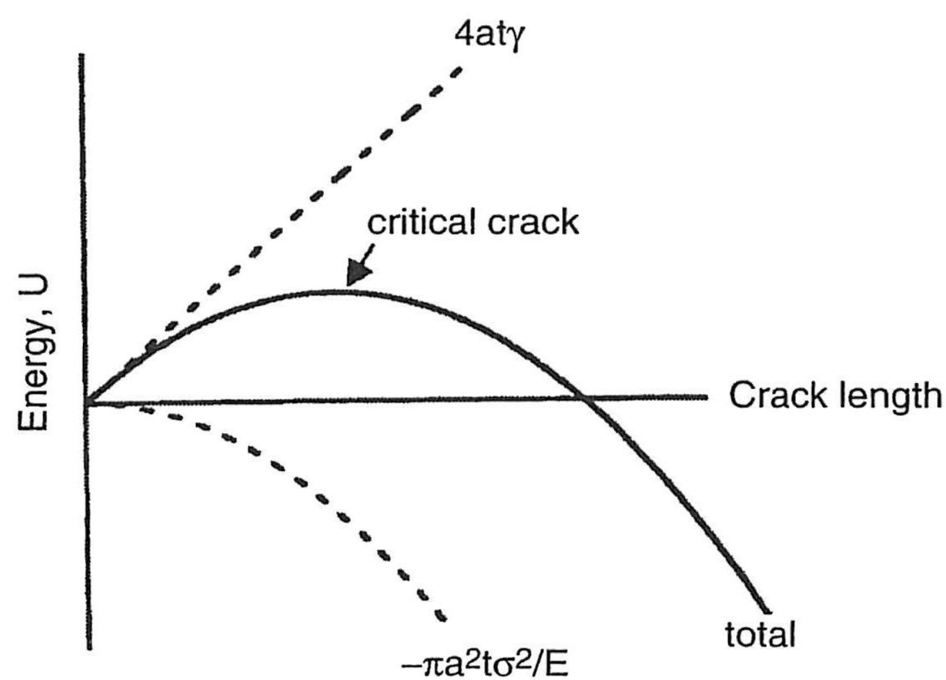
Equilibrium condition

$$\frac{d\Delta U}{da} = 4t\gamma_s - \frac{2\pi\, a^2 t \sigma^2}{E} = 0$$

Fracture stress

$$\sigma_f = \sqrt{\frac{2\gamma_s E}{\pi a}} = \sqrt{\frac{GE}{\pi a}}$$

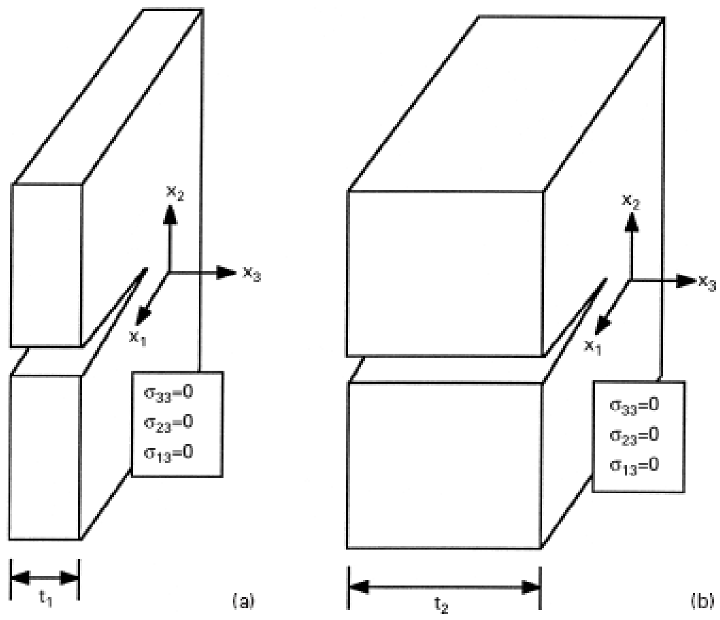
$$G = 2\gamma_s$$



Fracture stress between plane stress and plane strain

Plane stress

$$\sigma_f = \sqrt{\frac{GE}{\pi a}}$$



Plane strain

$$\sigma_f = \sqrt{\frac{GE}{(1-\nu^2)\pi a}}$$

U_{el}: elastic energy of body with crack
$U_{surface}$: surface energy of body with crack
σ: applied stress
a: one-half crack length
t: thickness
E: modulus of elasticity
γ: specific surface energy

Orowan theory considering plasticity in Griffith crack propagation (γ_p)

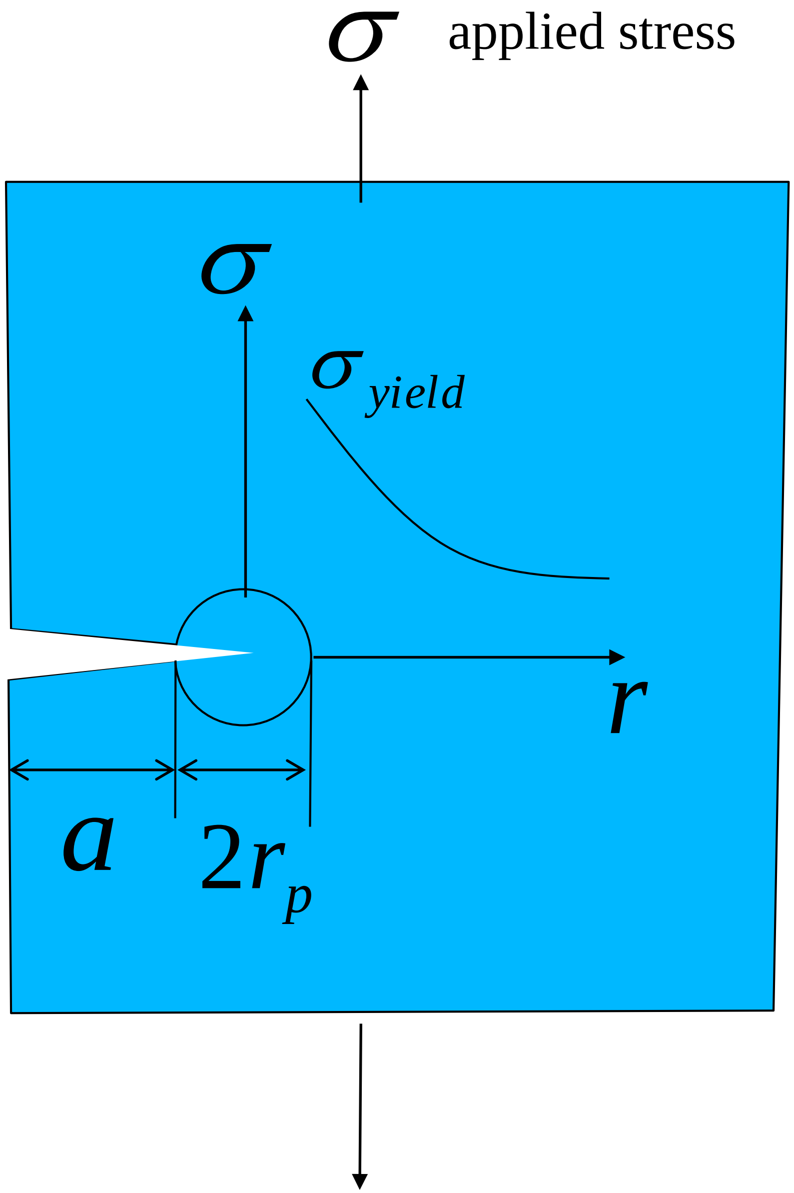
$$\sigma_f = \sqrt{\frac{GE}{\pi a}}$$

Including the plastic work in generating the fracture surface

$$G = 2(\gamma_s + \gamma_p)$$

Fracture toughness

$$K_c = \sqrt{EG_c} = \sigma_c \sqrt{\pi(a + r_p)}$$



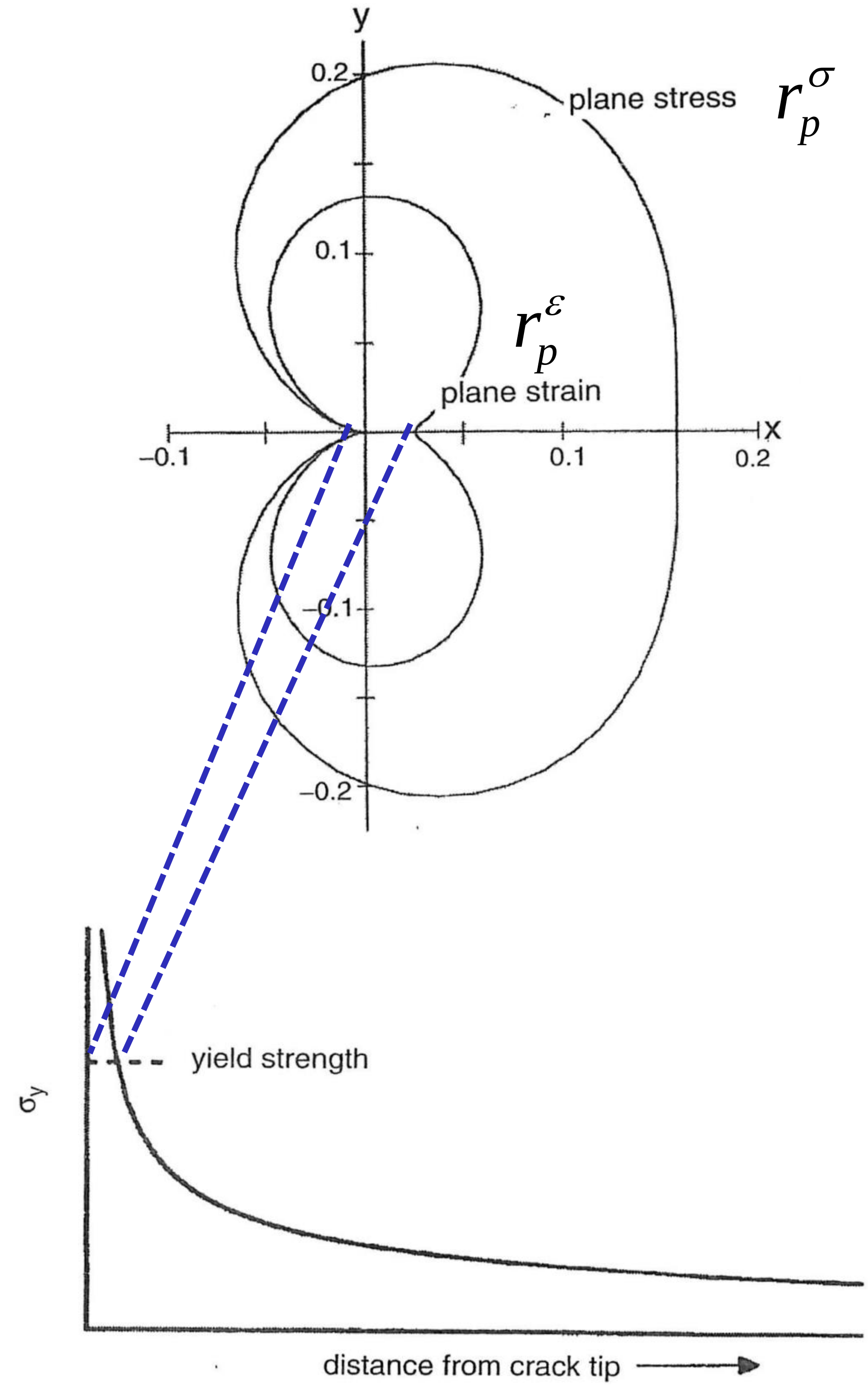
Plastic zone size between plane stress and plane strain

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

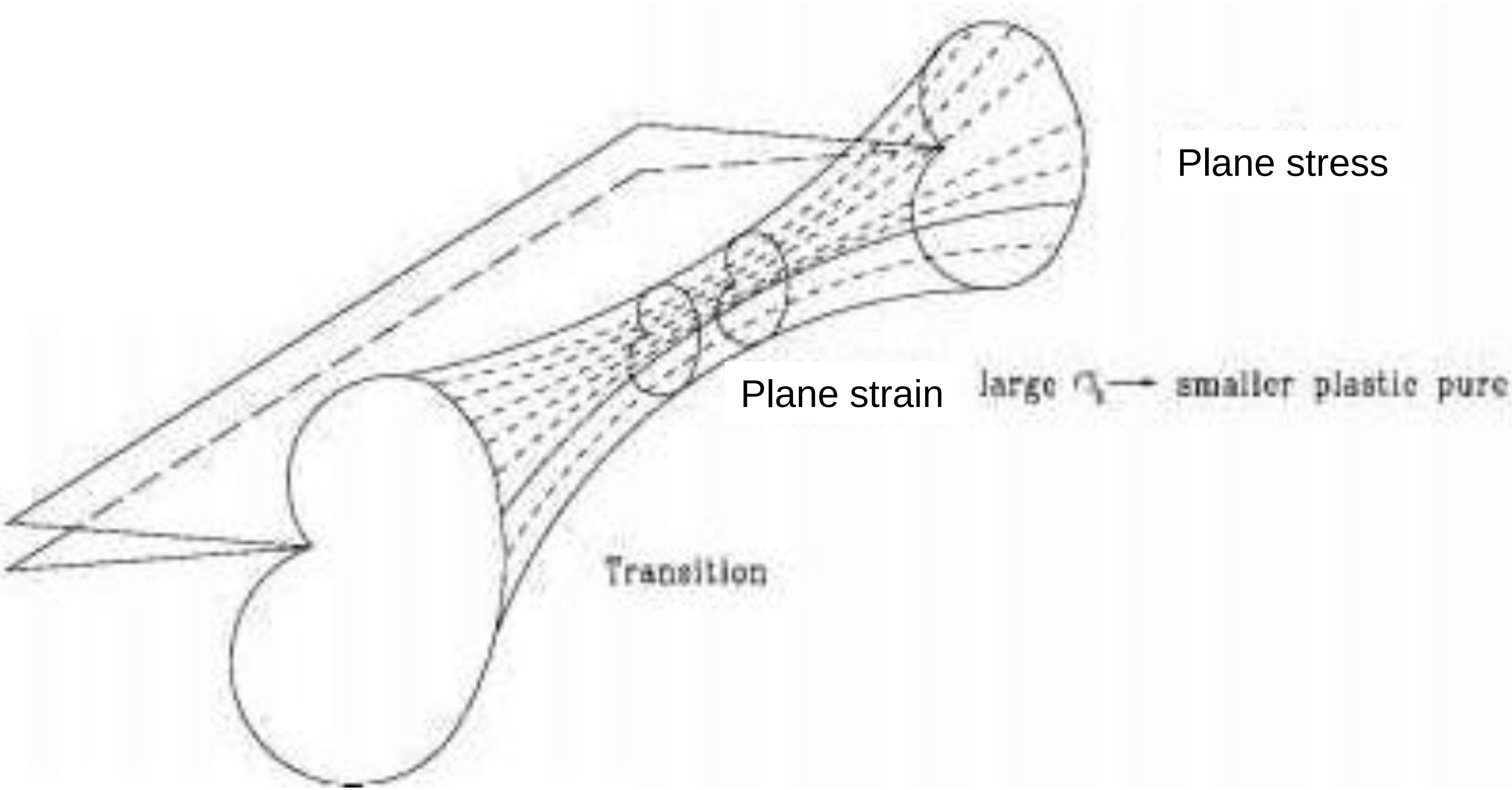
at $\theta = 0$ $\qquad \sigma_y = \frac{K}{\sqrt{2\pi r}}$

$$r_{\mathrm{p}} = \frac{\left(\frac{K_{\mathrm{c}}}{Y}\right)^2}{2\pi} \qquad\qquad r_{\mathrm{p}} = \frac{\left(\frac{K_{\mathrm{c}}}{Y}\right)^2}{6\pi}$$

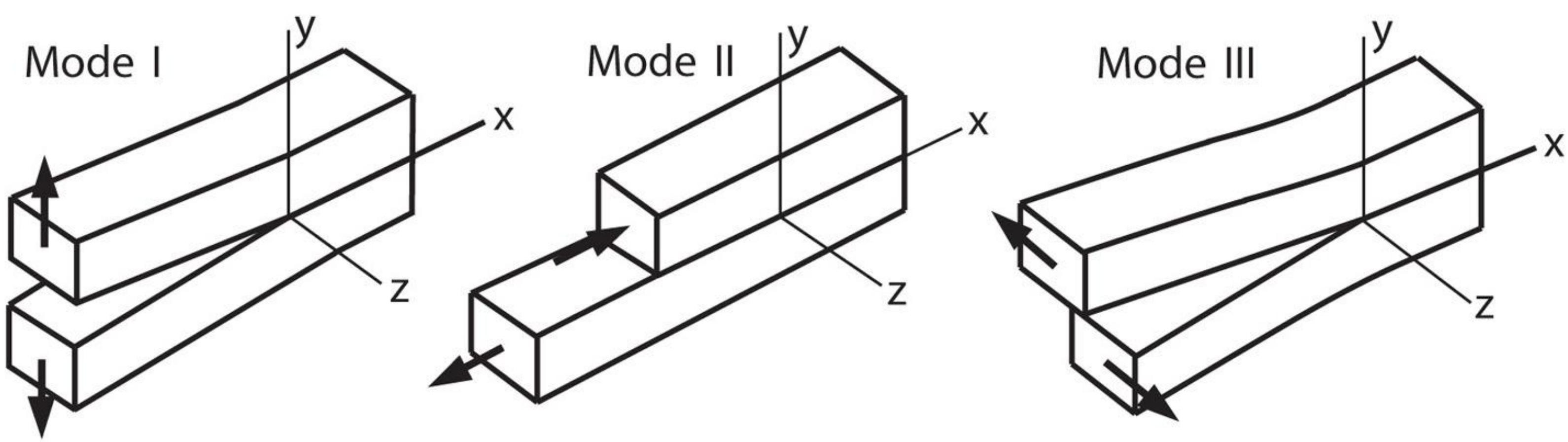
Plane stress $\qquad\qquad$ Plane strain



Plastic zone size through the crack tip



Facture modes and Deformation fileds



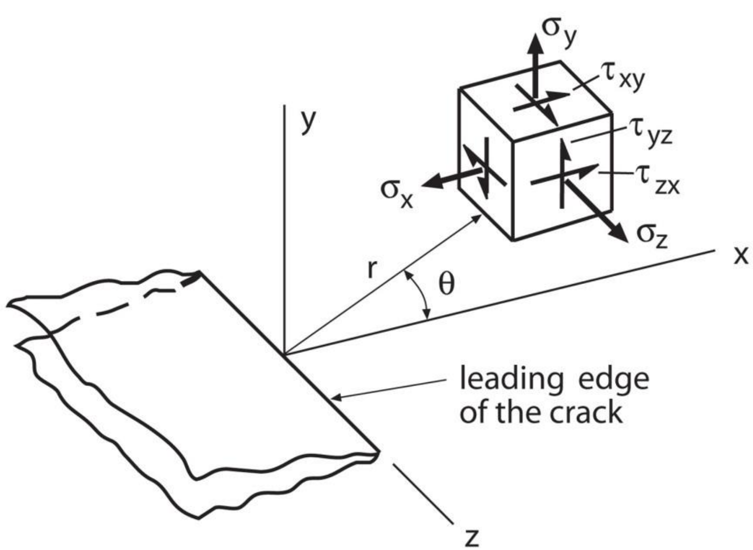
I. Opening or tensile mode

II. Sliding or In-plane shear mode

III. Tearing or antiplane shear mode

Irwin's fracture analysis (stress approach)

Irwin proposed the stress state around an Infinitely sharp crack in a semi-infinite elastic solid



$$\sigma_x = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_z = 0$$ plane stress condition

$$\sigma_z = \upsilon\left(\sigma_x + \sigma_y\right)$$ plane strain condition

$$\tau_{yz} = \tau_{zx} = 0$$

Definition of Stress intensity factor

Stress intensity factor in a semi-infinite body is given:

$$K = \sigma\sqrt{\pi a}$$

Stress intensity factor for finite body is given:

$$K = \sigma\sqrt{\pi a} \cdot f$$

f depends on the specimen geometry
and is >1 for small crack

Fracture occurs when K reaches a critical value, K_c, fracture toughness.

$$\sigma_f = \frac{K_c}{f(\pi a)^{1/2}} \qquad f = 1 \qquad K_c = \sqrt{EG_c}$$

Comparison between k_t and K

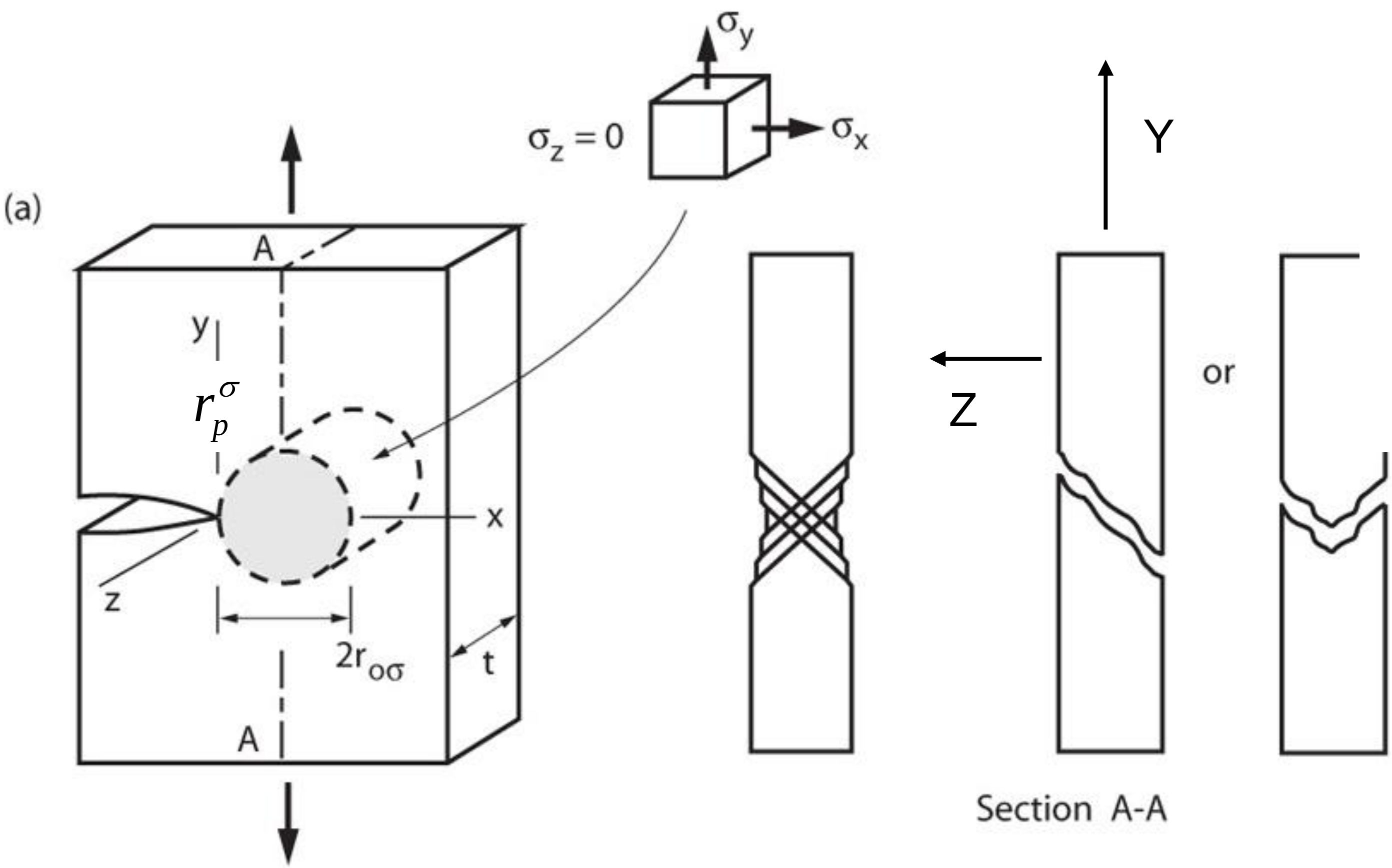
K (the stress intensity factor): provides a complete description of the state of stress, strain and displacement over some region of the body, is dependent of the crack length and the geometry of the body.

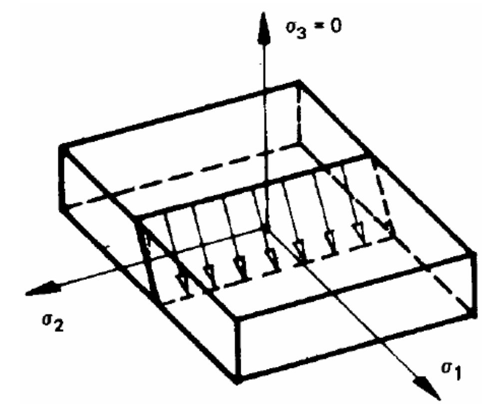
$$K = \sigma \cdot \sqrt{\pi a} \cdot f\left(\frac{a}{w}\right)$$

k_t (the stress concentration factor): determines the magnitude of the maximum stress at a single point.

$$k_t = 2\sqrt{\frac{a}{\rho}}$$

Slant fracture : Plane stress





$$\varepsilon_x \neq 0$$

$$\varepsilon_y \neq 0$$

$$\varepsilon_z \neq 0$$

$$\tau_{\max} = ?$$

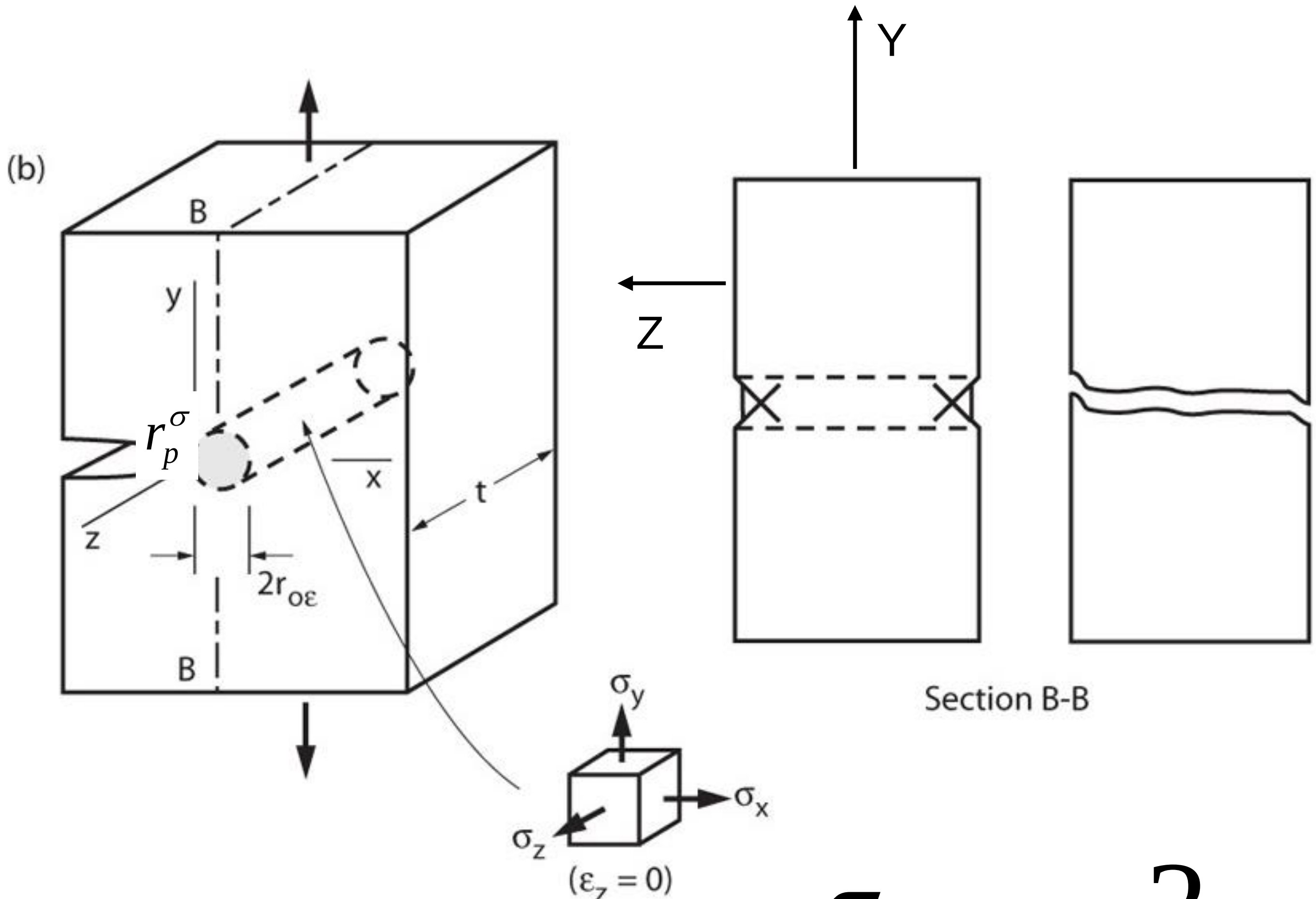
$$\sigma_y > \sigma_x$$

$$\sigma_z = 0$$

Remarks:
Biaxial stress state

Maximal shear occurs at Y-Z plane

Flat fracture: Plane strain



$$\sigma_x \neq 0$$

$$\sigma_y \neq 0$$

$$\sigma_z = \nu\left(\sigma_x + \sigma_y\right)$$

$$\varepsilon_x \neq 0$$

$$\varepsilon_y \neq 0$$

$$\varepsilon_z = 0$$

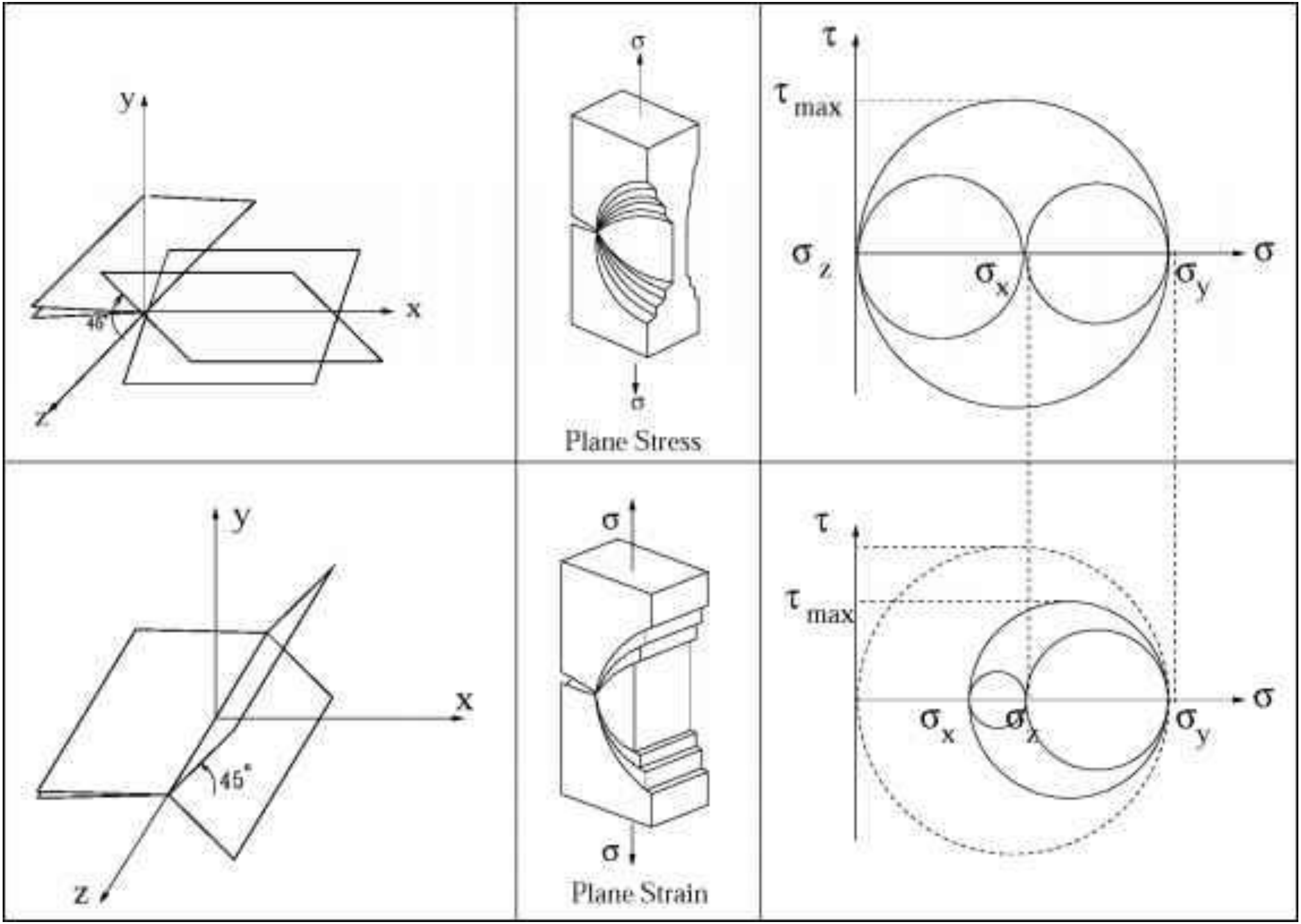
$$\tau_{max} = ?$$

Maximal shear occurs at X-Y plane $\sigma_y > \sigma_z > \sigma_x$

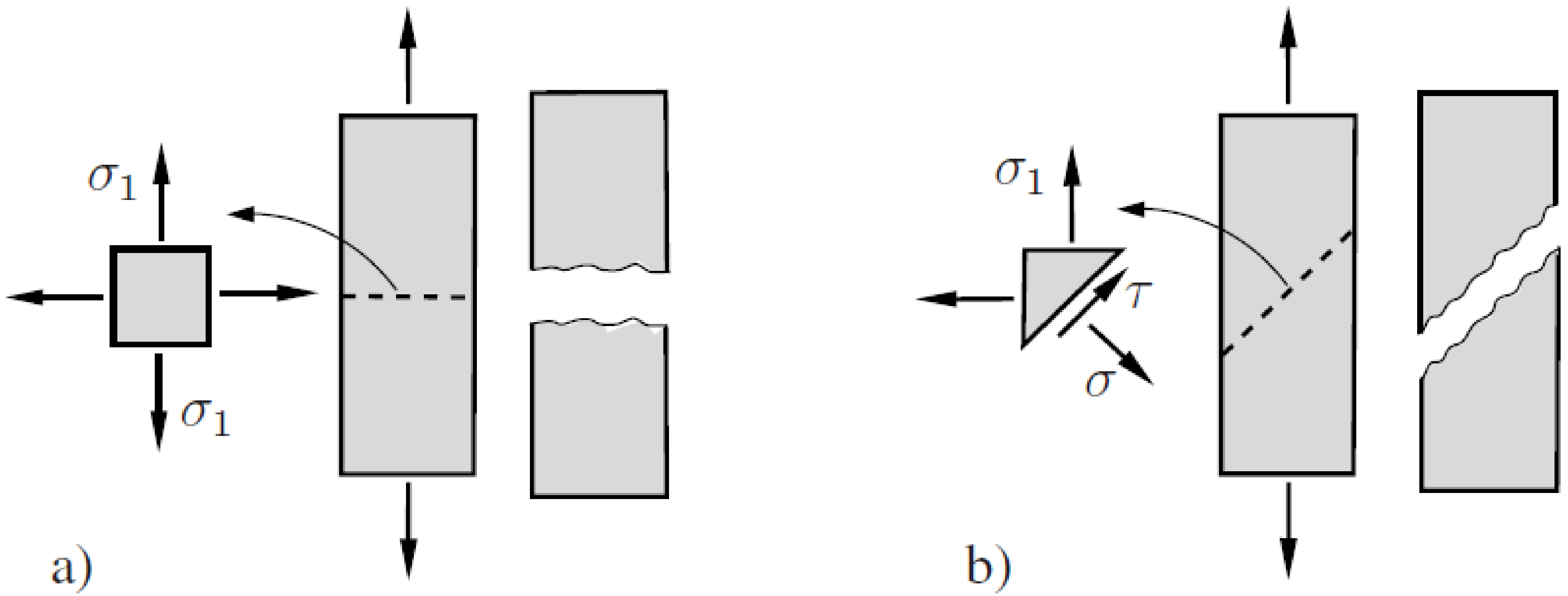
Remarks:

1. The triaxial stress state of plane strain reduces the plastic zone size in comparison to the plane stress zone size.
2. The triaxial stress state is pronounced at the boundary between the plastic and elastic zones.

Fracture plane: Plane stress and Plane strain



Fracture surface



Effect of thickness on K_c

$$K_c = \sqrt{EG_c}$$

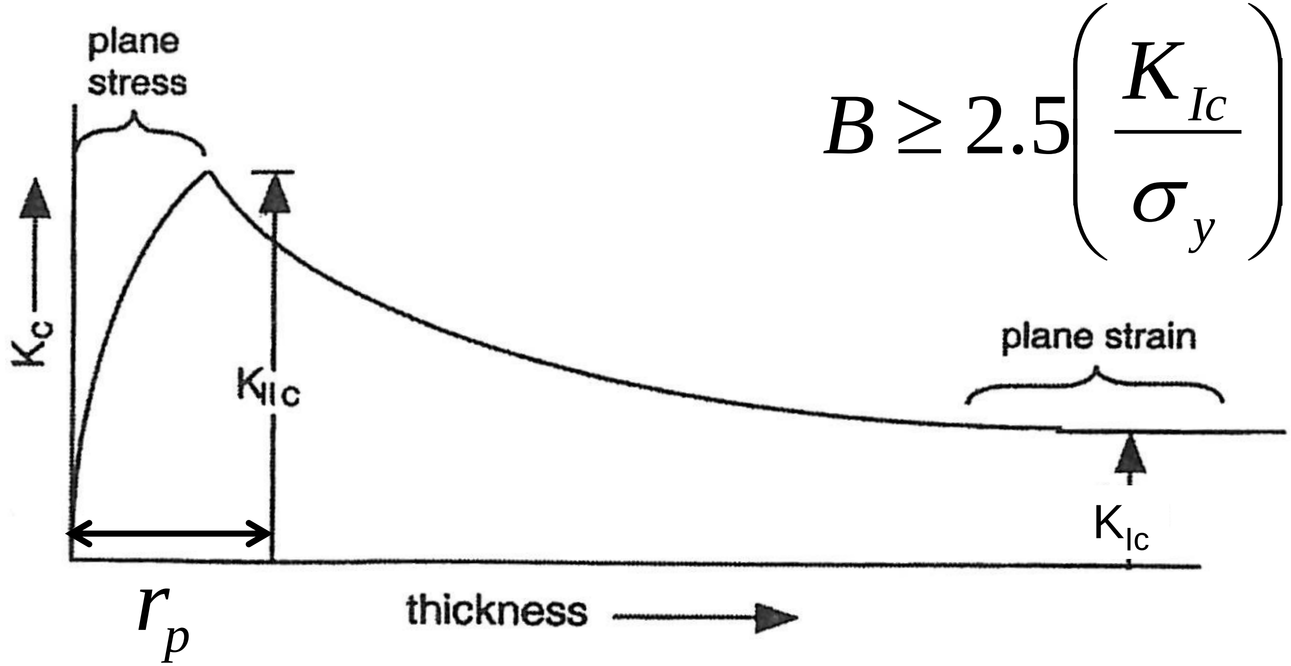
$$G_c = 2(\gamma_s + \gamma_p)$$

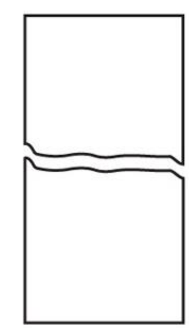
$$t \le 2 \cdot r_p$$

The thickness of the specimen should be much greater than the radius of the plastic zone for plane stress:

$$a \ge 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

$$B \ge 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$





Yield strength: metal and ceramics

		σ_y (MPa)	σ_{ts} (MPa)
Metals			
Ferrous	Cast Irons	215 - 790	350 - 1000
	High Carbon Steels	400 - 1155	550 - 1640
	Medium Carbon Steels	305 - 900	410 - 1200
	Low Carbon Steels	250 - 395	345 - 580
	Low Alloy Steels	400 - 1100	460 - 1200
	Stainless Steels	170 - 1000	480 - 2240
Non-ferrous	Aluminium Alloys	30 - 500	58 - 550
	Copper Alloys	30 - 500	100 - 550
	Lead Alloys	8 - 14	12 - 20
	Magnesium Alloys	70 - 400	185 - 475
	Nickel Alloys	70 - 1100	345 - 1200
	Titanium Alloys	250 - 1245	300 - 1625
	Zinc Alloys	80 - 450	135 - 520
Ceramics			
Glasses	Borosilicate Glass (*)	*264* - *384*	22 - 32
	Glass Ceramic (*)	*750* - *2129*	62 - 177
	Silica Glass (*)	*1100* - *1600*	45 - 155
	Soda-Lime Glass (*)	*360* - *420*	31 - 35
Porous	Brick (*)	*50* - *140*	7 - 14
	Concrete, typical (*)	*32* - *60*	2 - 6
	Stone (*)	*34* - *248*	5 - 17
Technical	Alumina (*)	*690* *5500*	350 665
	Aluminium Nitride (*)	*1970* - *2700*	197 - 270
	Boron Carbide (*)	*2583* - *5687*	350 - 560
	Silicon (*)	*3200* - *3460*	160 - 180
	Silicon Carbide (*)	*1000* - *5250*	370 - 680
	Silicon Nitride (*)	*524* - *5500*	690 - 800
	Tungsten Carbide (*)	*3347* - *6833*	370 - 550

Yield strength: polymer

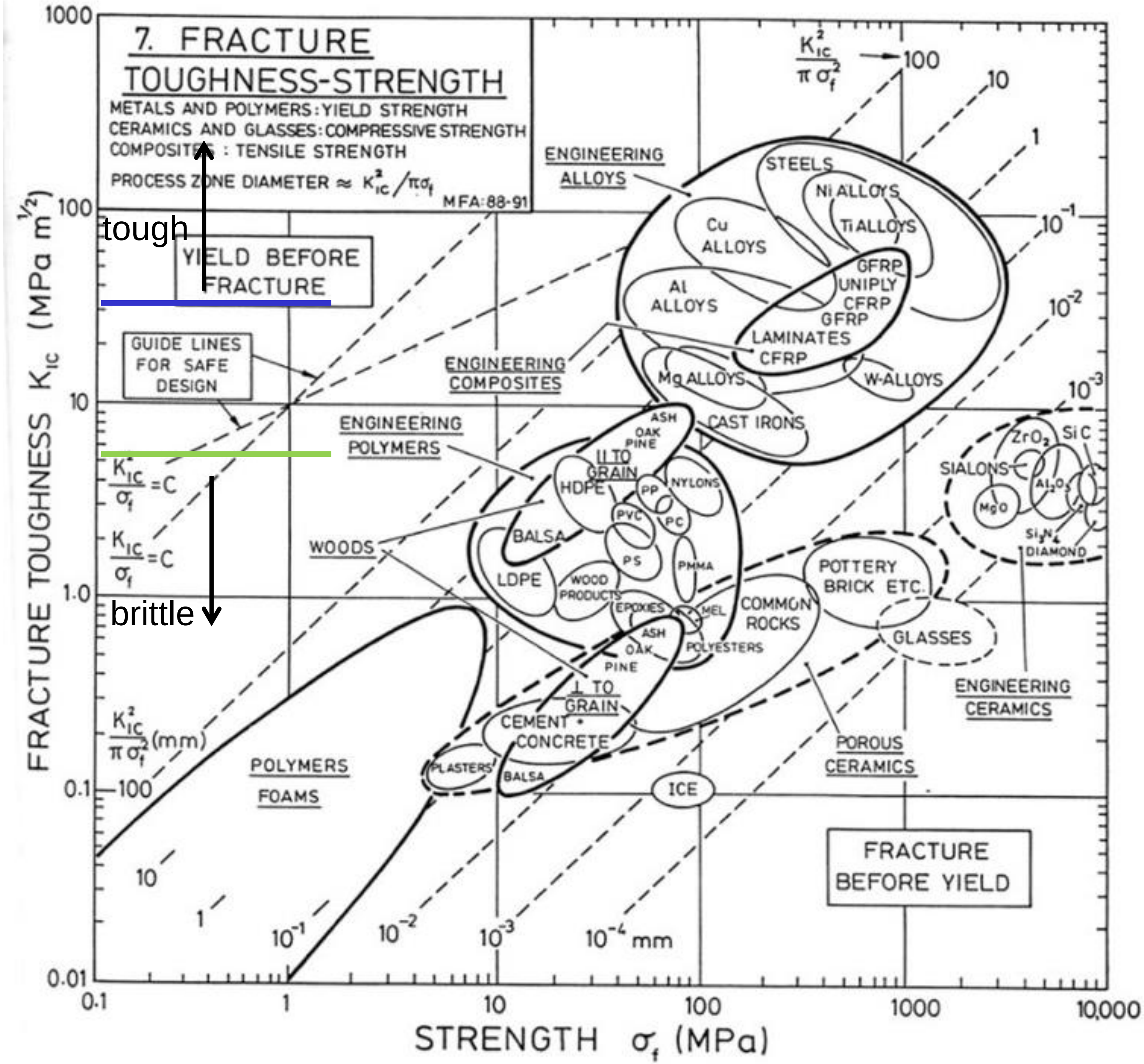
		σ_y (MPa)			σ_{ts} (MPa)		
Polymers [1]							
Elastomer	Butyl Rubber	2	-	3	5	-	10
	EVA	12	-	18	16	-	20
	Isoprene (IR)	20	-	25	20	-	25
	Natural Rubber (NR)	20	-	30	22	-	32
	Neoprene (CR)	3.4	-	24	3.4	-	24
	Polyurethane Elastomers (elPU)	25	-	51	25	-	51
	Silicone Elastomers	2.4	-	5.5	2.4	-	5.5
Thermoplastic	ABS	18.5	-	51	27.6	-	55.2
	Cellulose Polymers (CA)	25	-	45	25	-	50
	Ionomer (I)	8.3	-	15.9	17.2	-	37.2
	Nylons (PA)	50	-	94.8	90	-	165
	Polycarbonate (PC)	59	-	70	60	-	72.4
	PEEK	65	-	95	70	-	103
	Polyethylene (PE)	17.9	-	29	20.7	-	44.8
	PET	56.5	-	62.3	48.3	-	72.4
	Acrylic (PMMA)	53.8	-	72.4	48.3	-	79.6
	Acetal (POM)	48.6	-	72.4	60	-	89.6
	Polypropylene (PP)	20.7	-	37.2	27.6	-	41.4
	Polystyrene (PS)	28.7	-	56.2	35.9	-	56.5
	Polyurethane Thermoplastics (tpPU)	40	-	53.8	31	-	62
	PVC	35.4	-	52.1	40.7	-	65.1
	Teflon (PTFE)	15	-	25	20	-	30
Thermoset	Epoxies	36	-	71.7	45	-	89.6
	Phenolics	27.6	-	49.7	34.5	-	62.1
	Polyester	33	-	40	41.4	-	89.6

Fracture toughness: metal and ceramics

		K_{IC} (MPa$\sqrt{m}$)
Metals		
Ferrous	Cast Irons	22 - 54
	High Carbon Steels	27 - 92
	Medium Carbon Steels	12 - 92
	Low Carbon Steels	41 - 82
	Low Alloy Steels	14 - 200
	Stainless Steels	62 - 280
Non-ferrous	Aluminium Alloys	22 - 35
	Copper Alloys	30 - 90
	Lead Alloys	5 - 15
	Magnesium Alloys	12 - 18
	Nickel Alloys	80 - 110
	Titanium Alloys	14 - 120
	Zinc Alloys	10 - 100
Ceramics		
Glasses	Borosilicate Glass	0.5 - 0.7
	Glass Ceramic	1.4 - 1.7
	Silica Glass	0.6 - 0.8
	Soda-Lime Glass	0.55 - 0.7
Porous	Brick	1 - 2
	Concrete, typical	0.35 - 0.45
	Stone	0.7 - 1.5
Technical	Alumina	3.3 4.8
	Aluminium Nitride	2.5 - 3.4
	Boron Carbide	2.5 - 3.5
	Silicon	0.83 - 0.94
	Silicon Carbide	2.5 - 5
	Silicon Nitride	4 - 6
	Tungsten Carbide	2 - 3.8

Fracture toughness: polymer

		K_{IC} (MPa$\sqrt{m}$)
Polymers [1]		
Elastomer	Butyl Rubber	0.07 - 0.1
	EVA	0.5 - 0.7
	Isoprene (IR)	0.07 - 0.1
	Natural Rubber (NR)	0.15 - 0.25
	Neoprene (CR)	0.1 - 0.3
	Polyurethane Elastomers (elPU)	0.2 - 0.4
	Silicone Elastomers	0.03 - 0.5
Thermoplastic	ABS	1.19 - 4.30
	Cellulose Polymers (CA)	1 - 2.5
	Ionomer (I)	1.14 - 3.43
	Nylons (PA)	2.22 - 5.62
	Polycarbonate (PC)	2.1 - 4.60
	PEEK	2.73 - 4.30
	Polyethylene (PE)	1.44 - 1.72
	PET	4.5 - 5.5
	Acrylic (PMMA)	0.7 - 1.6
	Acetal (POM)	1.71 - 4.2
	Polypropylene (PP)	3 - 4.5
	Polystyrene (PS)	0.7 - 1.1
	Polyurethane Thermoplastics (tpPU)	1.84 - 4.97
	PVC	1.46 - 5.12
	Teflon (PTFE)	1.32 - 1.8
Thermoset	Epoxies	0.4 - 2.22
	Phenolics	0.79 - 1.21
	Polyester	1.09 - 1.70



fracture considerations. (*Adapted from M. F. Ashby, Materials Selection in Mechanical Design, Pergamon Press, Oxford, 1992.*)

Fracture toughness testing

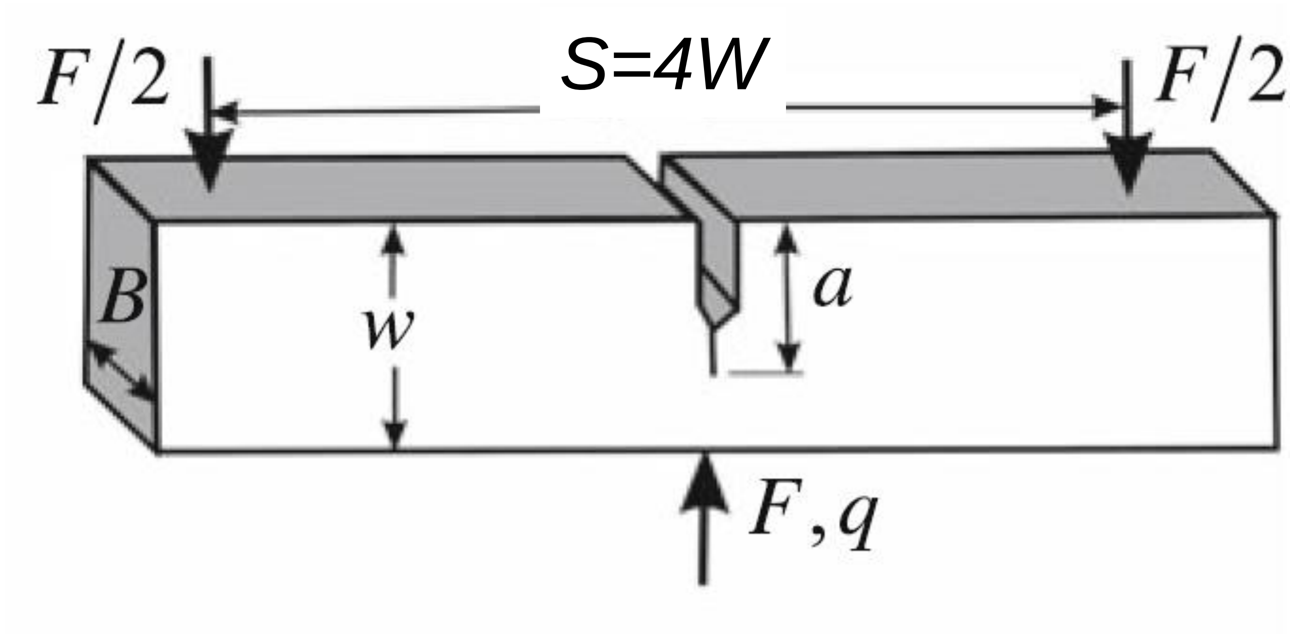
The following are the fracture toughness parameters commonly obtained from testing

- **K (stress intensity factor)** can be considered as a **stress-**based estimate of fracture toughness. K depends on geometry (the flaw depth, together with a geometric function, which is given in test standards for each test specimen geometry).

- **CTOD (crack-tip opening displacement)** can be considered as a **strain-**based estimate of fracture toughness. However, it can be separated into elastic and plastic components. The elastic part of CTOD is derived from the stress intensity factor, K. The plastic component is derived from the crack mouth opening displacement (measured using a clip gauge).

- **J (J-integral)** is an **energy-**based estimate of fracture toughness. It can be separated into elastic and plastic components. As with CTOD, the elastic component is based on K, while the plastic component is derived from the plastic area under the force-displacement curve.

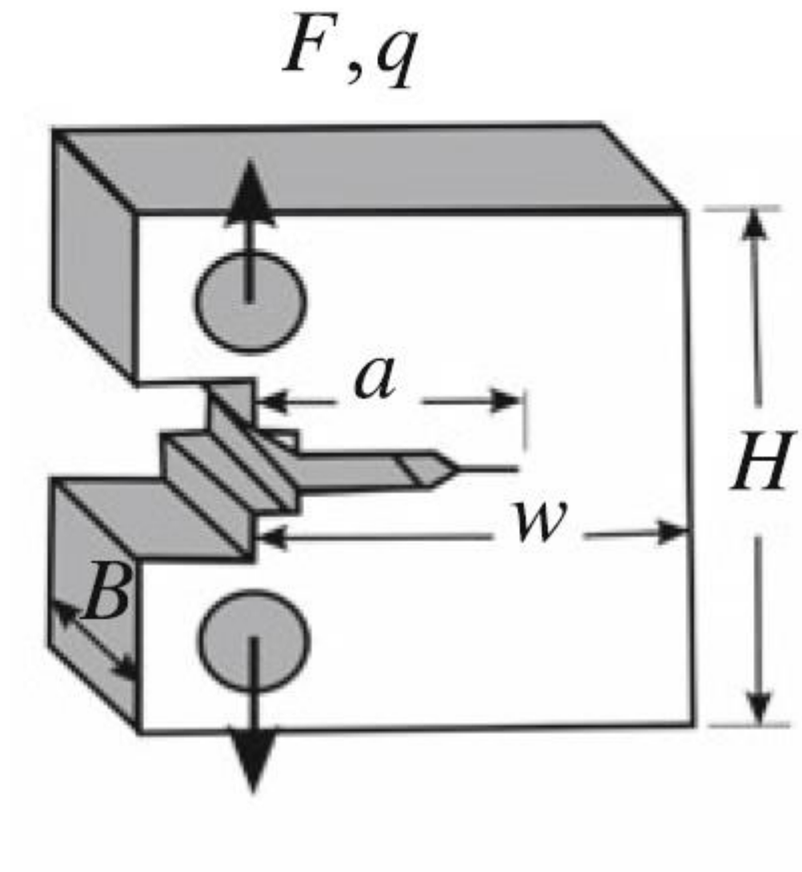
Plane-strain fracture testing of metals:
single edge notch bend (SENB or three-point bend)



$$B \geq 2.5\left(\frac{K_{IC}}{\sigma_y}\right)^2$$

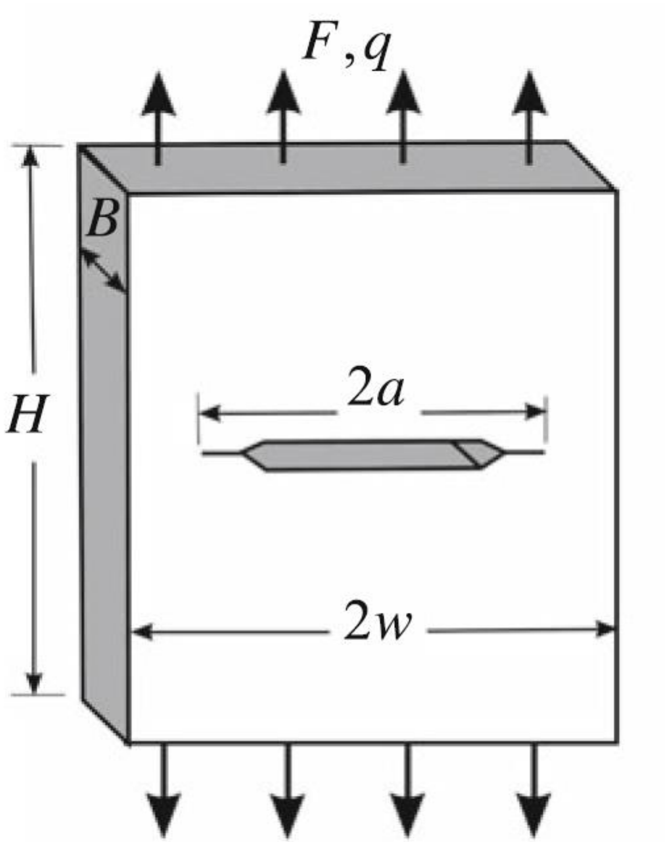
$$K_I = \frac{FS}{BW^{3/2}}\begin{bmatrix} 2.9(a/w)^{1/2} - 4.6(a/w)^{3/2} + 21.8(a/w)^{3/2} \\ -37.6(a/w)^{7/2} + 38.7(a/w)^{9/2} \end{bmatrix}$$

Plane-strain fracture testing of metals:
compact tension-off centre



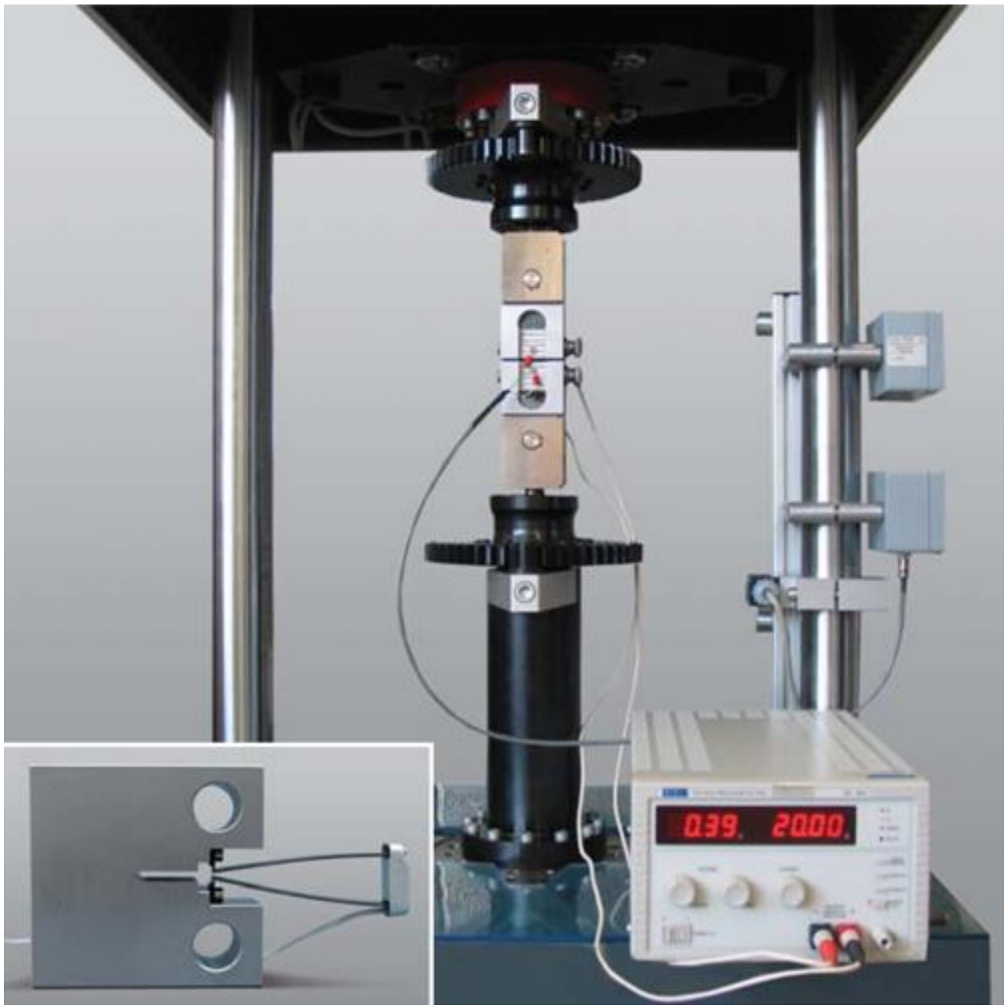
$$K_I = \frac{FS}{BW^{1/2}}\begin{bmatrix} 29.6(a/w)^{1/2} - 185.5(a/w)^{3/2} + 655.7(a/w)^{3/2} \\ -1017(a/w)^{7/2} + 63.9(a/w)^{9/2} \end{bmatrix}$$

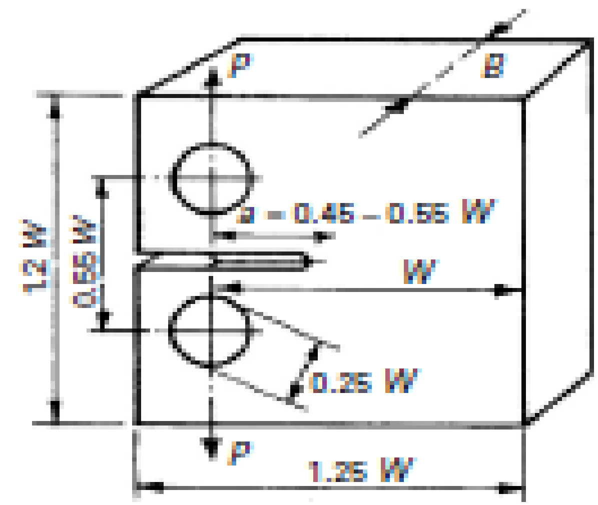
Plane-strain fracture testing of metals: Centre notched



$$K_I = \sigma\sqrt{\pi a}\left(\sec\frac{\pi a}{W}\right)^{1/2}$$

Measurement of fracture toughness





Measurement of fracture toughness