Mechanical Behaviour of Materials

Chapter 05-1

Dislocation: Basics

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Theoretical strength of crystals

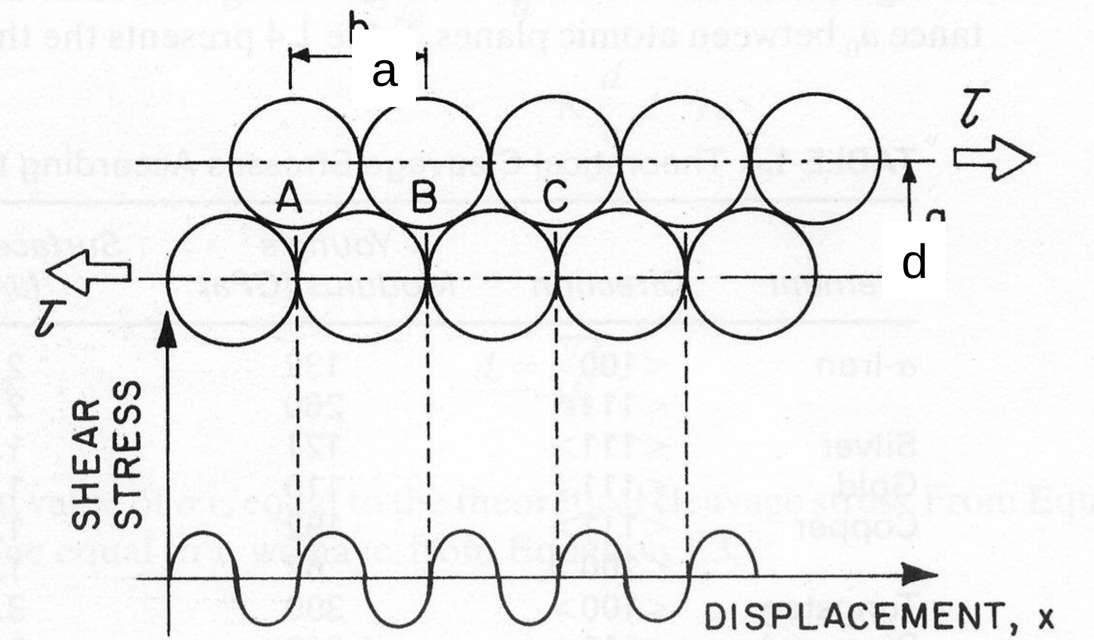


Figure 1.33 Stress required to shear a crystal.

According to Frenkel

$$\tau = \tau_{max} \sin \frac{2\pi x}{a}$$

$$\tau \approx \tau_{max} \frac{2\pi x}{a}$$

$$\tau = G \frac{x}{d}$$

$$\tau_{max} = \frac{Ga}{2\pi d}$$

$$d \approx a$$

$$\tau_{max} \approx \frac{G}{2\pi}$$

Is theoretical strength of crystals possible to achieve?

Why are the theoretical cleavage and shear strengths of materials possible to reach?

Table 9.1. Critical shear stress for slip in several materials

		Critical shear stress (MPa)	
Metal	**Purity %**	**Experiment**	**Theory**
copper	>99.9	1.0	414
silver	99.99	0.6	285
cadmium	99.996	0.58	207
iron	99.9	∼7.0	740

Note: There is considerable scatter caused by experimental variables, particularly purity.

Is theoretical strength of crystals possible to achieve?

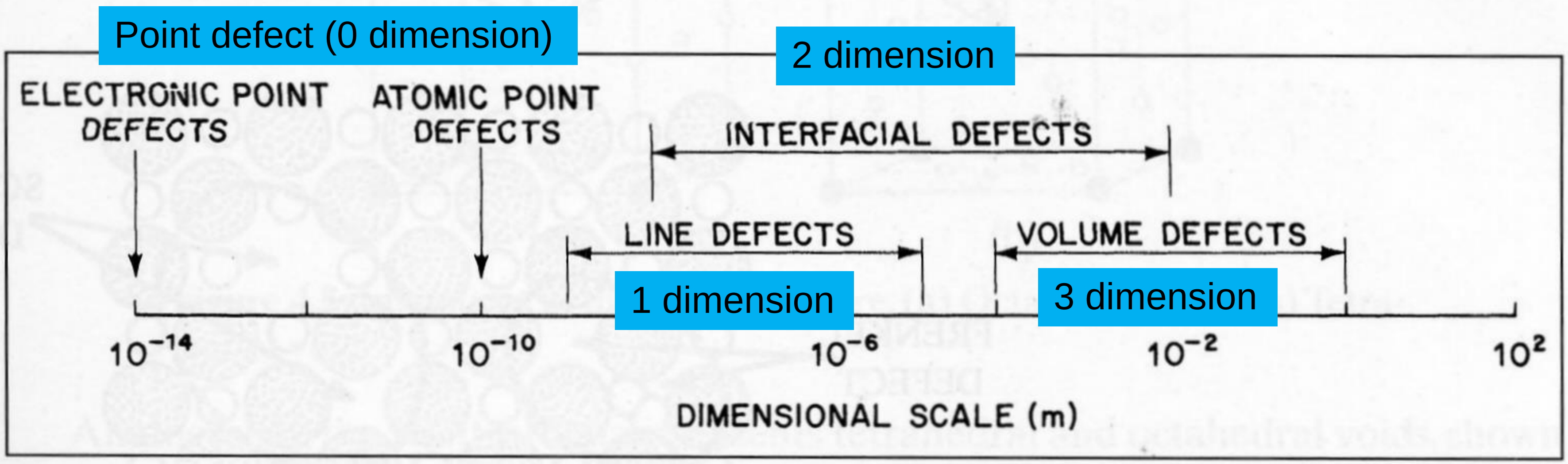
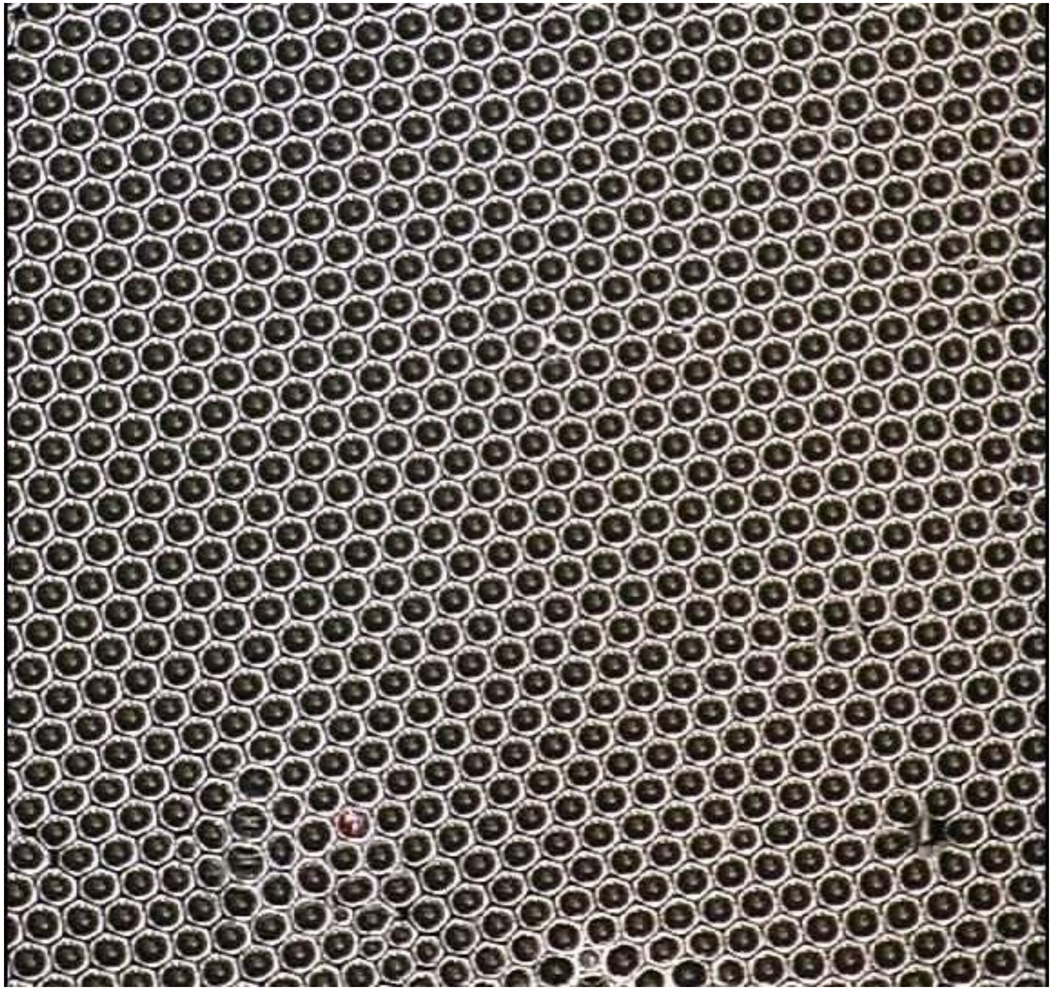


Figure 4.1 Dimensional ranges of different classes of defects.

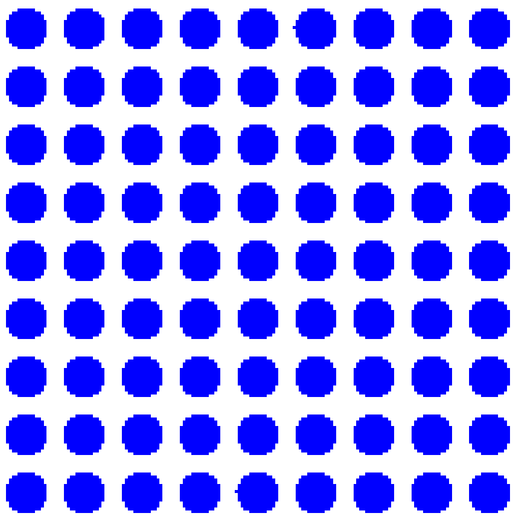
Why are the theoretical cleavage and shear strengths of materials possible to reach?

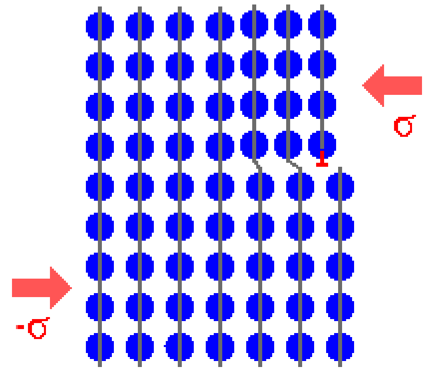
What kind of defects?



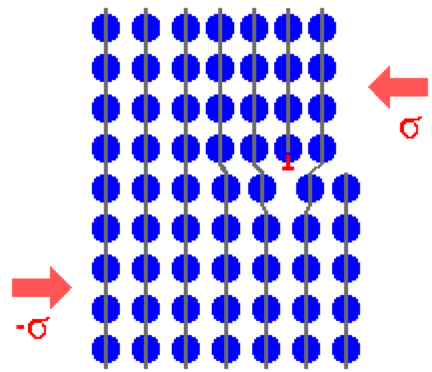
Is theoretical strength of crystals possible to achieve?

Dislocation movement and plastic deformation

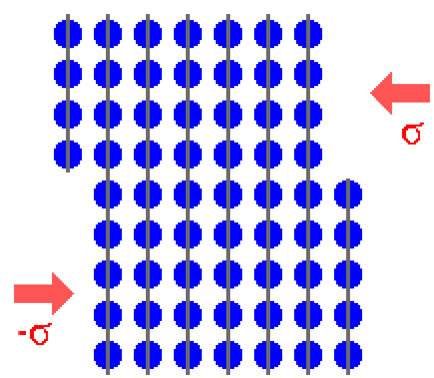




Generation of an edge dislocation
by a *shear* stress

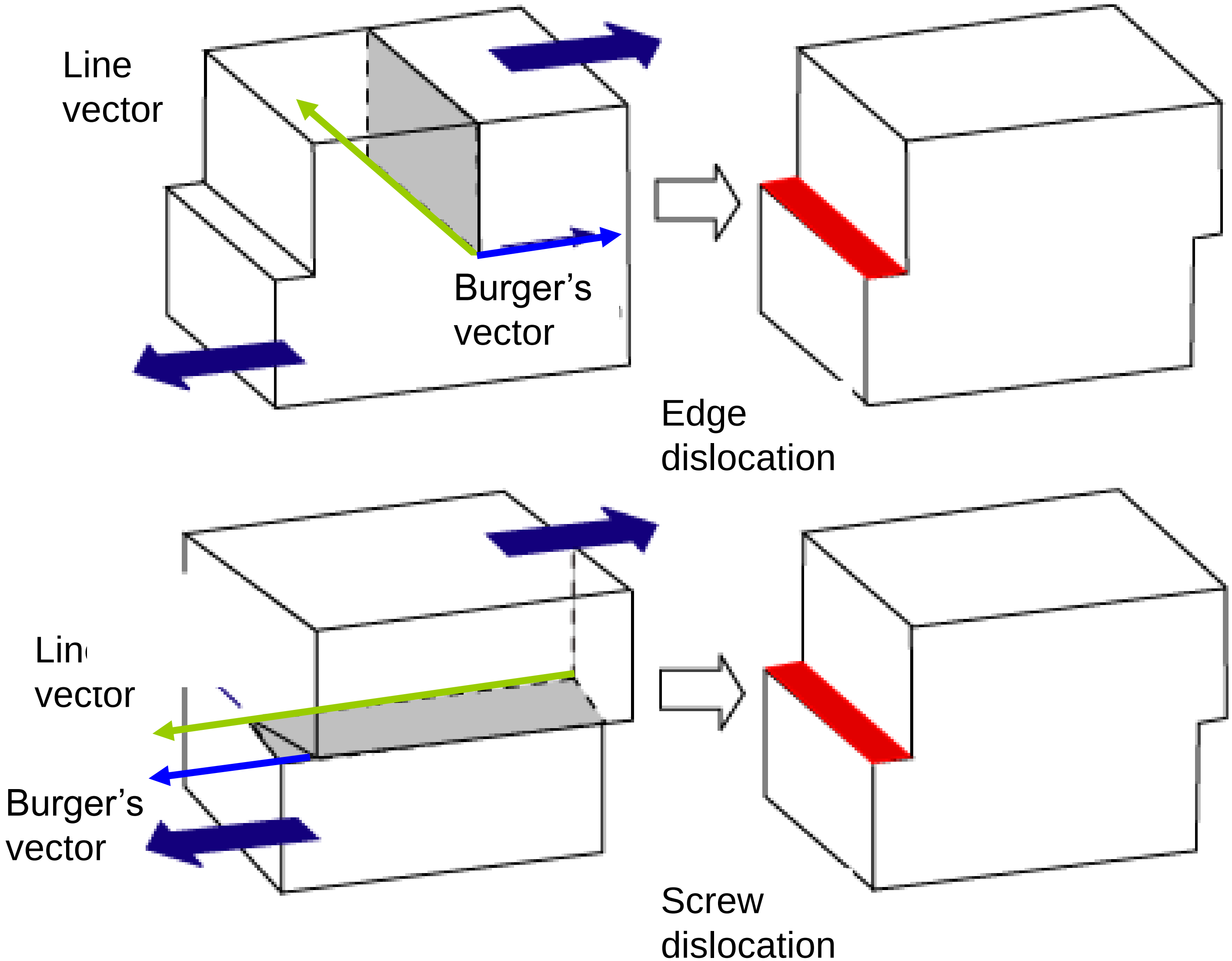


Movement of the dislocation
through the crystal



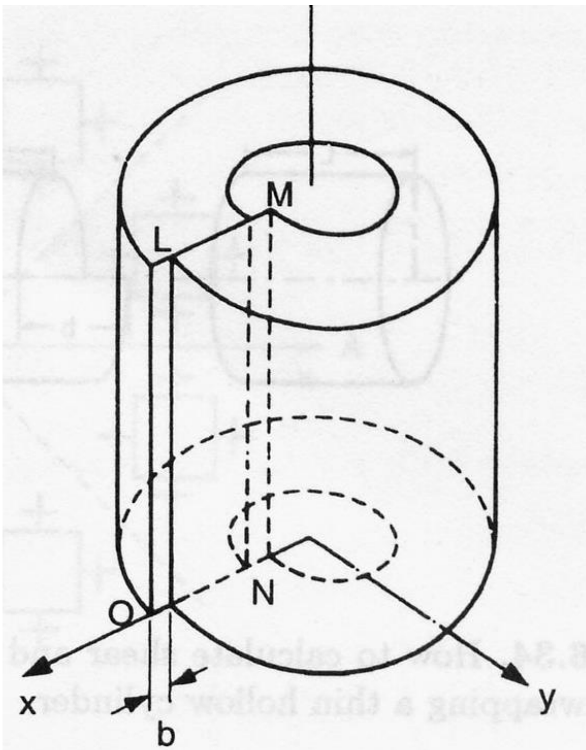
Shift of the upper half of the crystal
after the dislocation emerged

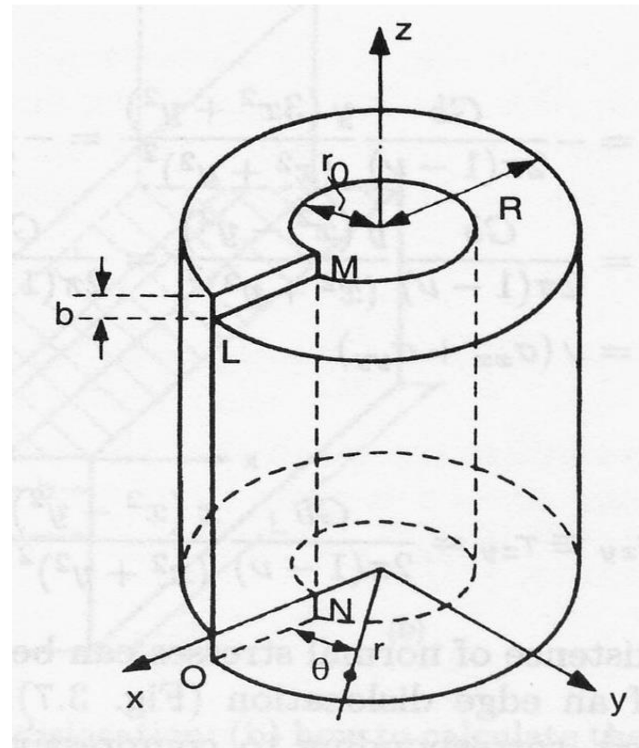
Definition of edge and screw dislocations



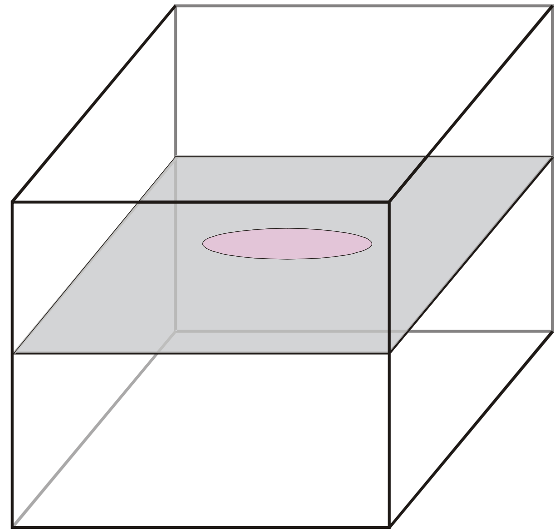
http://www.youtube.com/watch?v=z3MzDiyLtWc&feature=endscreen

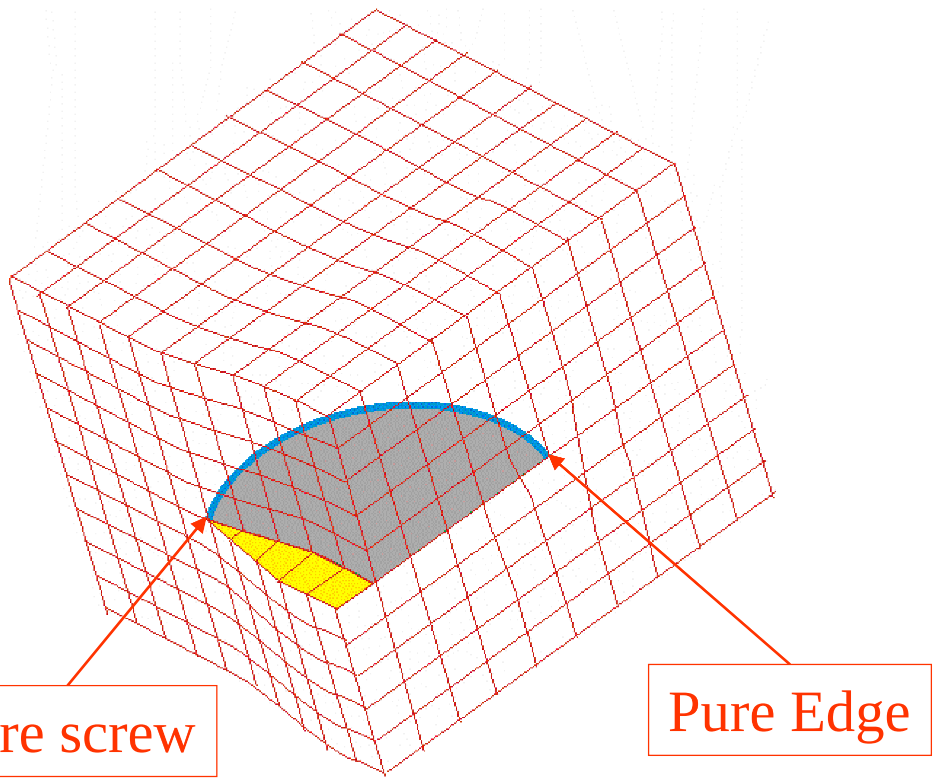
Definition of edge and screw dislocations



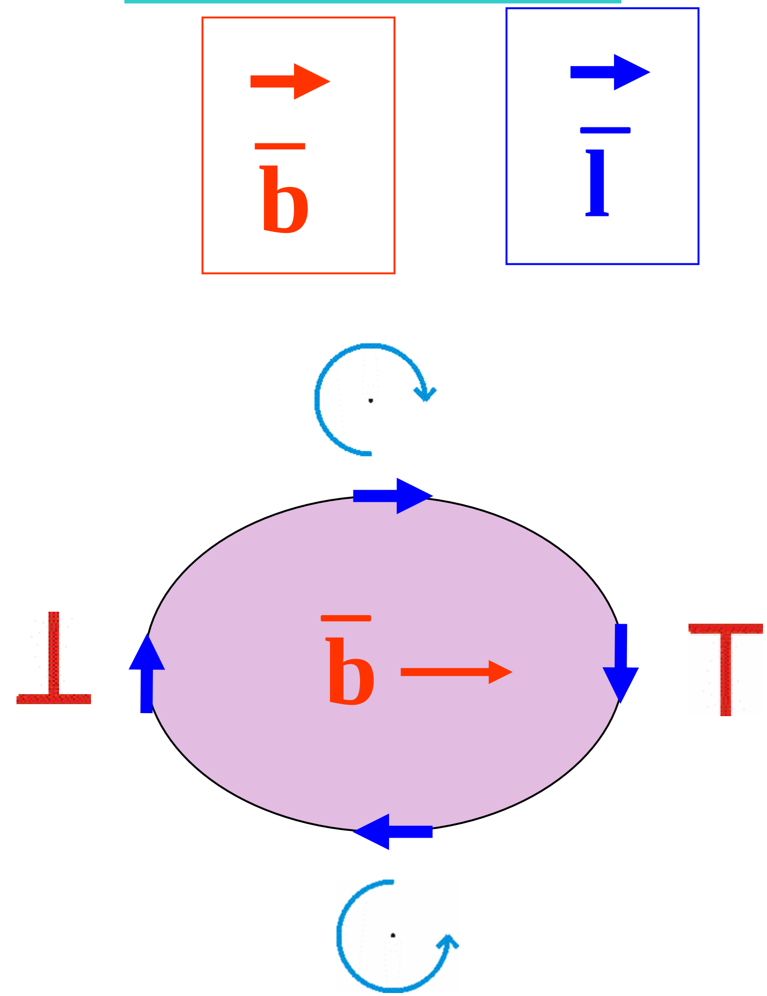


Mixing dislocations





Mixed dislocations



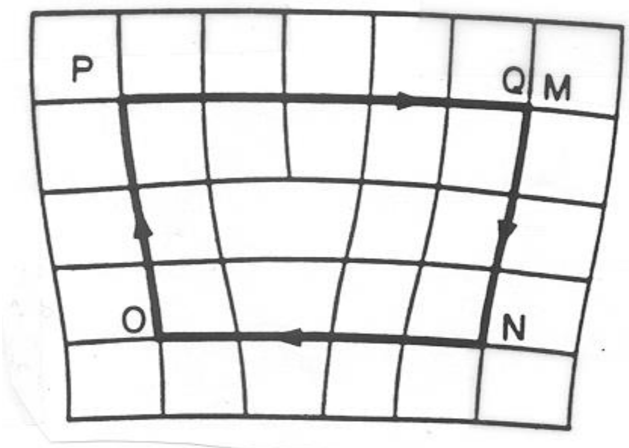
Summary

Dislocation Property	Type of dislocation	
	Edge	Screw
Relation between dislocation line (**l**) and **b**	$\perp$	$\parallel$
Slip direction	$\parallel$ to **b**	$\parallel$ to **b**
Direction of dislocation line movement relative to **b**	$\parallel$	$\perp$
Process by which dislocation may leave slip plane	climb	Cross-slip

What is Burgers vector?

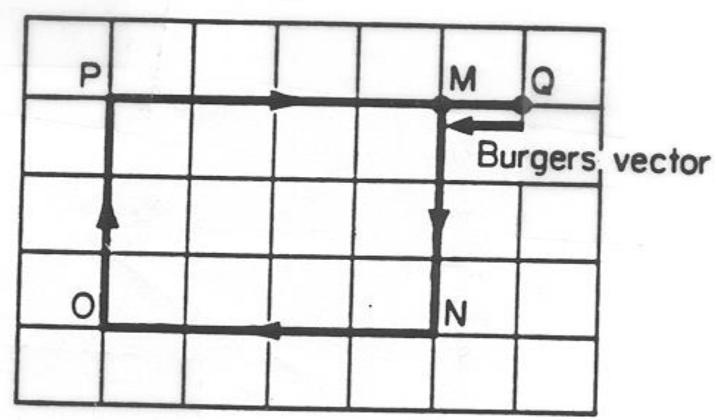
To determine the Burgers vector of a dislocation in a two-dimensional primitive square lattice, proceed as follows:

Trace around the end of the dislocation plane to form a closed loop. Record the number of lattice vectors travelled along each side of the loop (shown here by the numbers in the boxes):



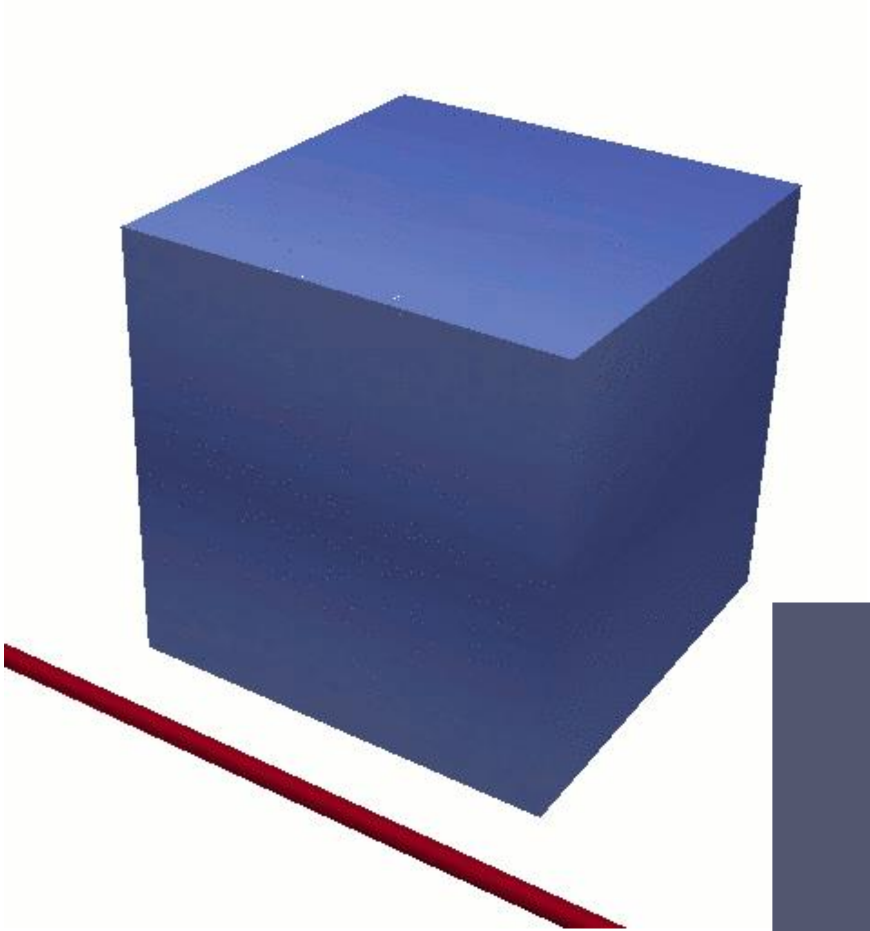
In a perfect lattice, trace out the same path, moving the same number of lattice vectors along each direction as before. This loop will not be complete, and the closure failure is the Burgers vector:

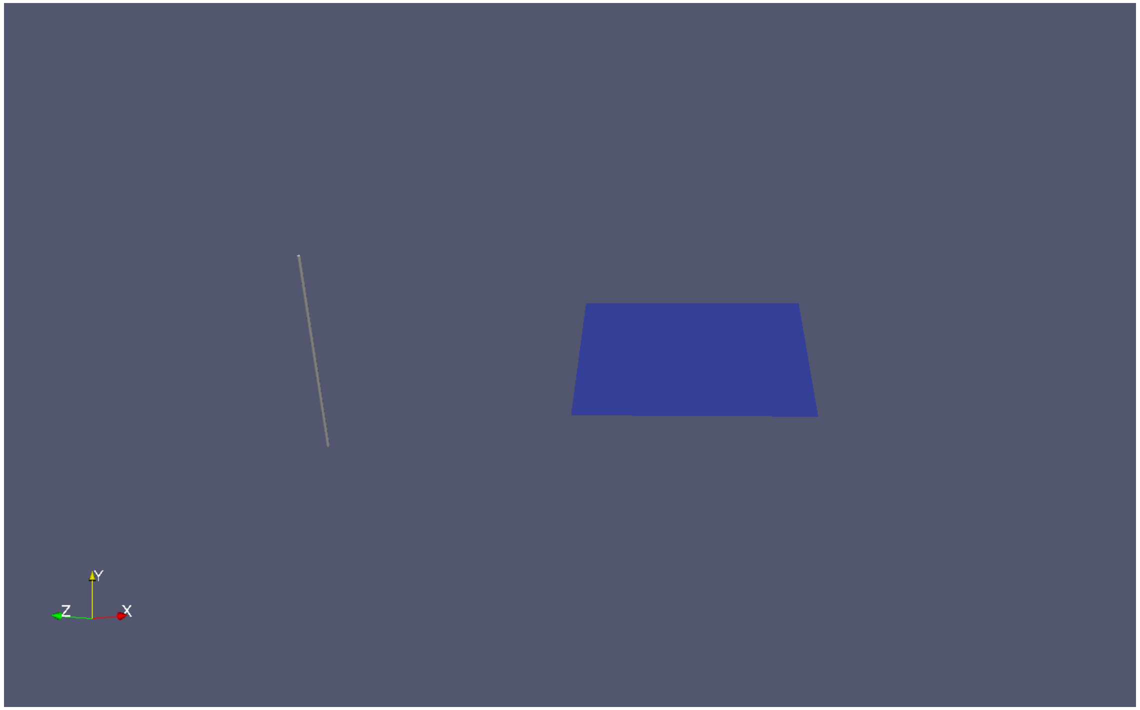
RHFS:
Right Hand Finish to Start
convention



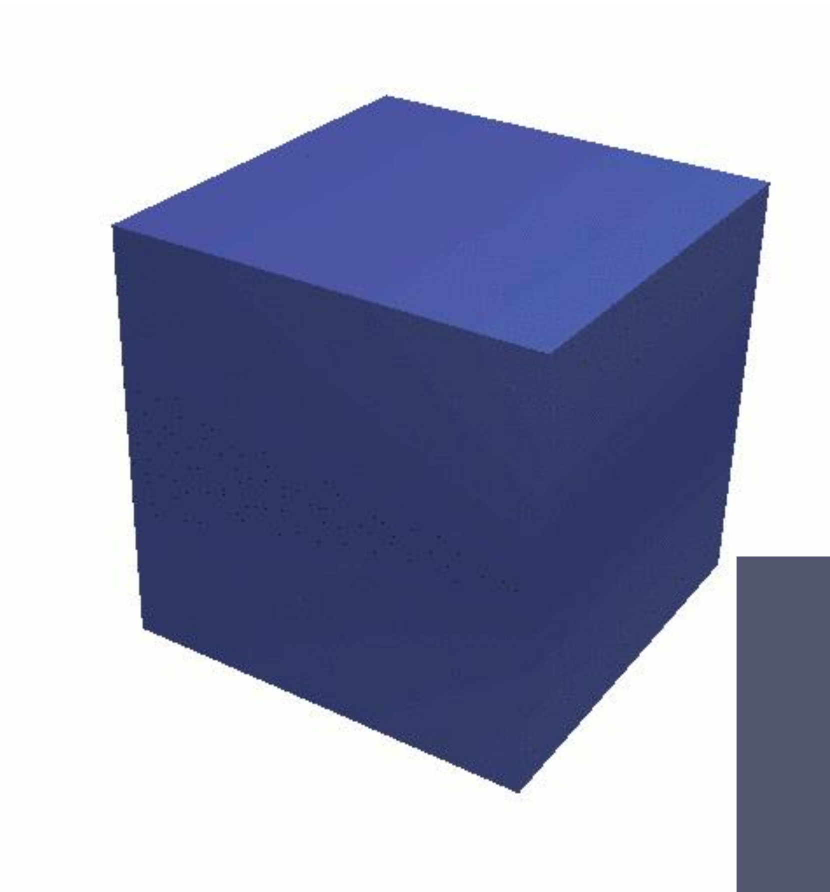
Home work: Burgers vector for fcc, bcc and hcp

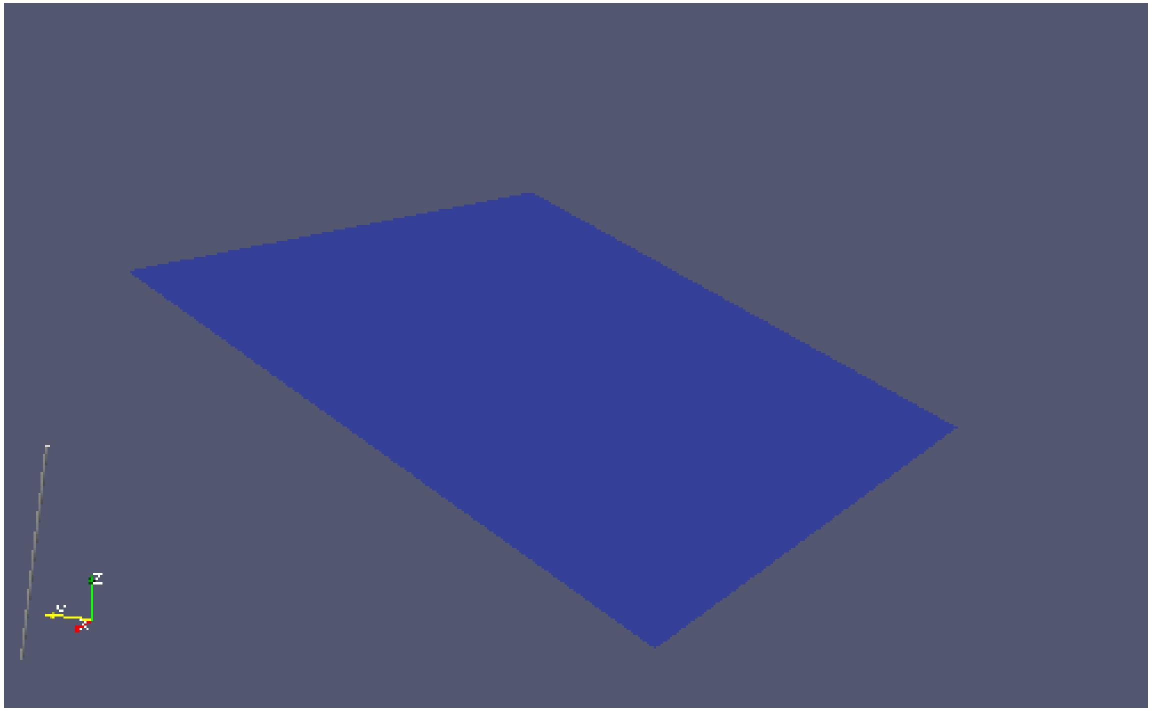
Movement of edge dislocation





Movement of screw dislocation



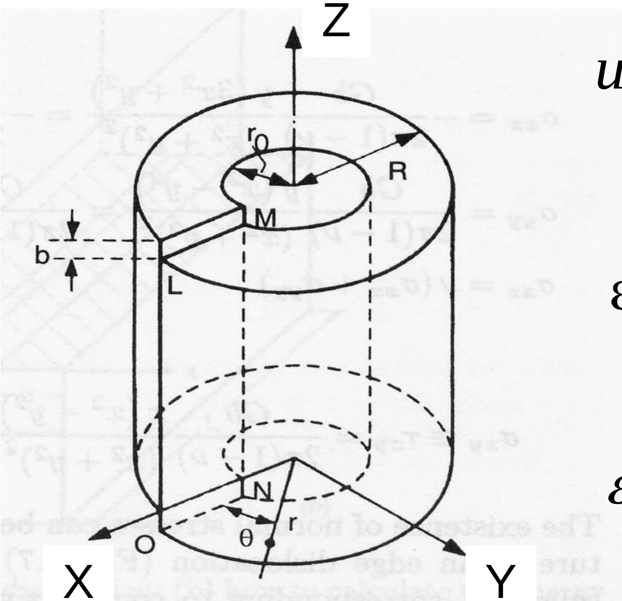


Home work: Differentiation of $\dfrac{\partial \tan^{-1}\frac{y}{x}}{\partial x}$ and $\dfrac{\partial \tan^{-1}\frac{y}{x}}{\partial y}$

$$\frac{\partial \tan^{-1}\frac{y}{x}}{\partial x} = \frac{-\frac{y}{x^2}}{1+\left(\frac{y}{x}\right)^2}$$

$$\frac{\partial \tan^{-1}\frac{y}{x}}{\partial y} = \frac{\frac{1}{x}}{1+\left(\frac{y}{x}\right)^2}$$

Stress field of a screw dislocation



$$u_z = \frac{\theta}{2\pi} b = \frac{b}{2\pi} \tan^{-1} \frac{y}{x}$$

$$\varepsilon_{ij} = \begin{pmatrix} 0 & 0 & \varepsilon_{xz} \\ 0 & 0 & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & 0 \end{pmatrix}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

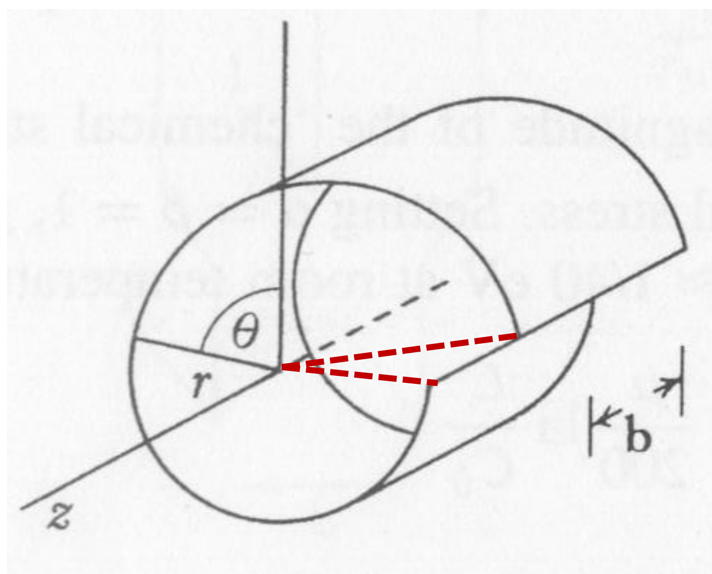
$$\varepsilon_{zz} = \frac{1}{2} \frac{\partial u_z}{\partial z} = 0$$

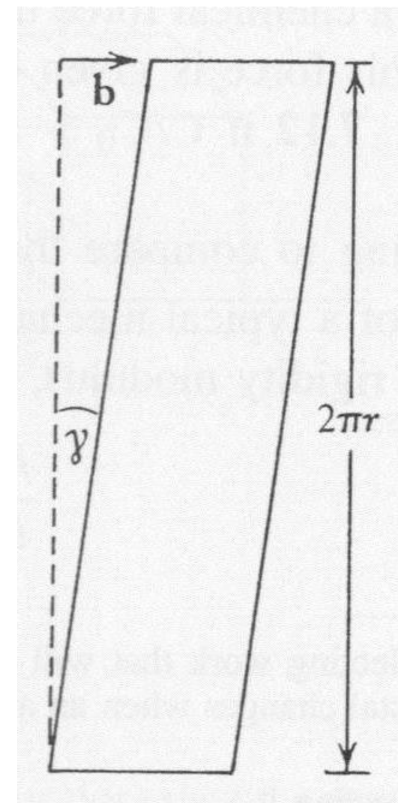
$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{xy} = 0$$

$$u_x = u = 0$$

$$u_y = v = 0$$

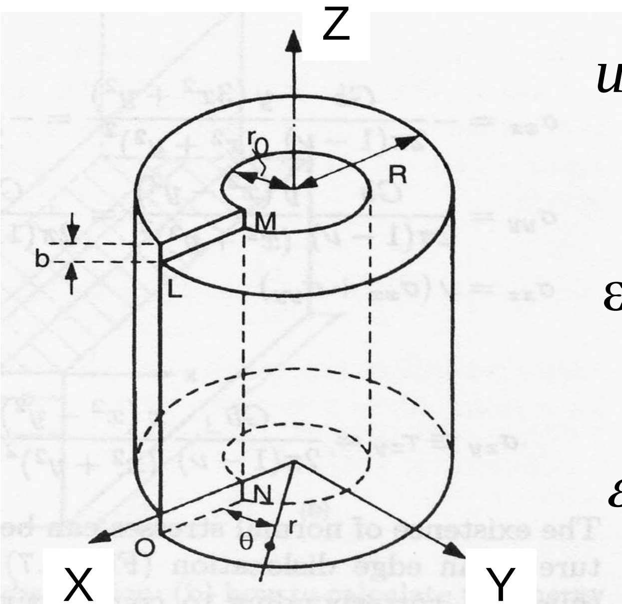
$$u_z = w \neq 0$$





Stress field of a screw dislocation

$$\sigma_{ij} = \begin{pmatrix} 0 & 0 & \sigma_{xz} \\ 0 & 0 & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & 0 \end{pmatrix}$$



$$u_z = \frac{\theta}{2\pi} b = \frac{b}{2\pi} \tan^{-1} \frac{y}{x}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{xy} = 0$$

$$\varepsilon_{yz} = \frac{1}{2}\frac{\partial u_z}{\partial y} = \frac{bx}{4\pi(x^2 + y^2)}$$

$$\sigma_{yz} = \frac{Gbx}{2\pi(x^2 + y^2)}$$

$$\varepsilon_{xz} = \frac{1}{2}\frac{\partial u_z}{\partial x} = -\frac{by}{4\pi(x^2 + y^2)}$$

$$\sigma_{xz} = \frac{-Gby}{2\pi(x^2 + y^2)}$$

$$u_x = 0$$

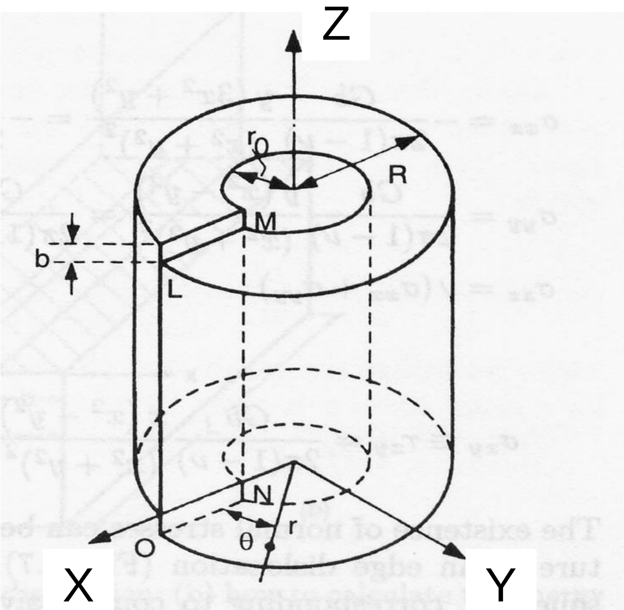
$$u_y = 0$$

$$u_z \neq 0$$

$$\varepsilon_{zz} = \frac{1}{2}\frac{\partial u_z}{\partial z} = 0$$

Stress field of a screw dislocation

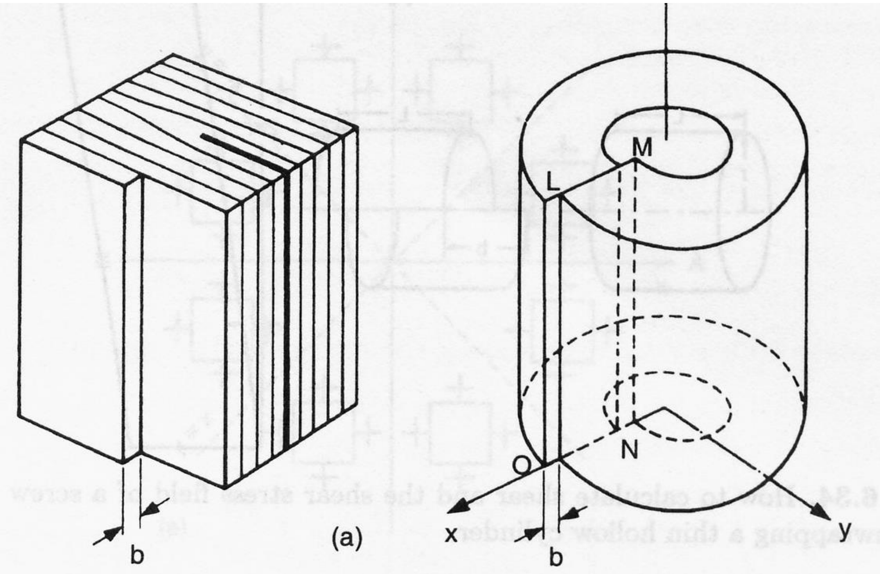
$$\sigma_{ij} = \begin{pmatrix} 0 & 0 & \sigma_{xz} \\ 0 & 0 & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & 0 \end{pmatrix}$$



$$\sigma_{yz} = \frac{Gbx}{2\pi(x^2+y^2)} = \frac{Gb}{2\pi}\frac{\cos\theta}{r}$$

$$\sigma_{xz} = \frac{Gby}{2\pi(x^2+y^2)} = \frac{Gb}{2\pi}\frac{\sin\theta}{r}$$

Stress field of a edge dislocation



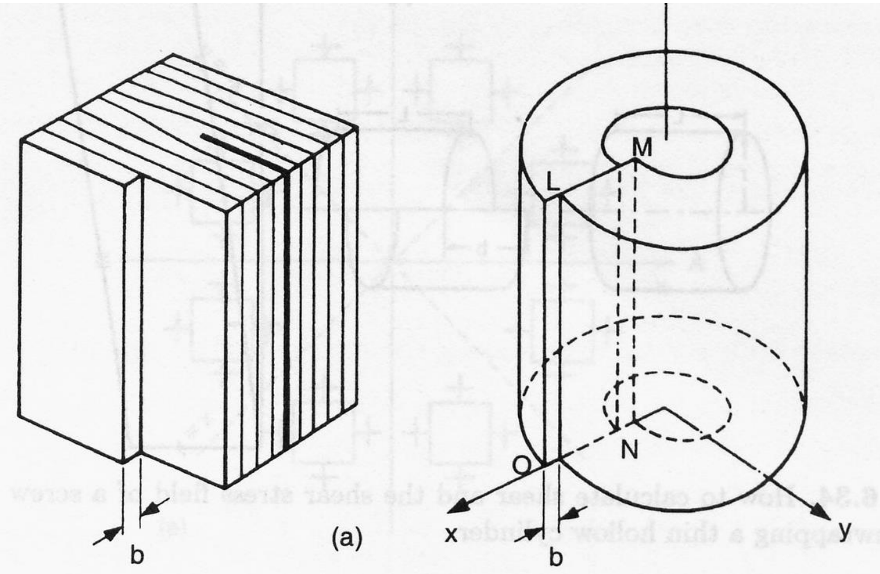
$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

$$u_x = \frac{b}{2\pi}\left[\tan^{-1}\frac{y}{x} + \frac{1}{2(1-\nu)}\cdot\frac{xy}{x^2+y^2}\right] = \frac{b}{2\pi}\left[\theta + \frac{\sin^2\theta}{4(1-\nu)}\right]$$

$$u_y = \frac{-b}{8\pi(1-\nu)}\left[(1-2\nu)\log\left(x^2+y^2\right) + \frac{x^2-y^2}{x^2+y^2}\right] = \frac{-b}{2\pi}\left[\frac{1-2\nu}{2(1-\nu)}\ln r + \frac{\cos 2\theta}{4(1-2\nu)}\right]$$

$$u_z = 0$$

Stress field of a edge dislocation



$$\sigma_{xx} = -\frac{Gb}{2\pi(1-\upsilon)} \frac{y(3x^2+y^2)}{(x^2+y^2)^2}$$

$$\sigma_{yy} = \frac{Gb}{2\pi(1-\upsilon)} \frac{y(x^2-y^2)}{(x^2+y^2)^2}$$

$$\sigma_{zz} = \upsilon(\sigma_{xx}+\sigma_{yy})$$

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

$$\sigma_{xy} = \sigma_{yx}$$

$$= \frac{Gb}{2\pi(1-\upsilon)} \frac{x(x^2-y^2)}{(x^2+y^2)^2}$$

Stress field of a edge dislocation

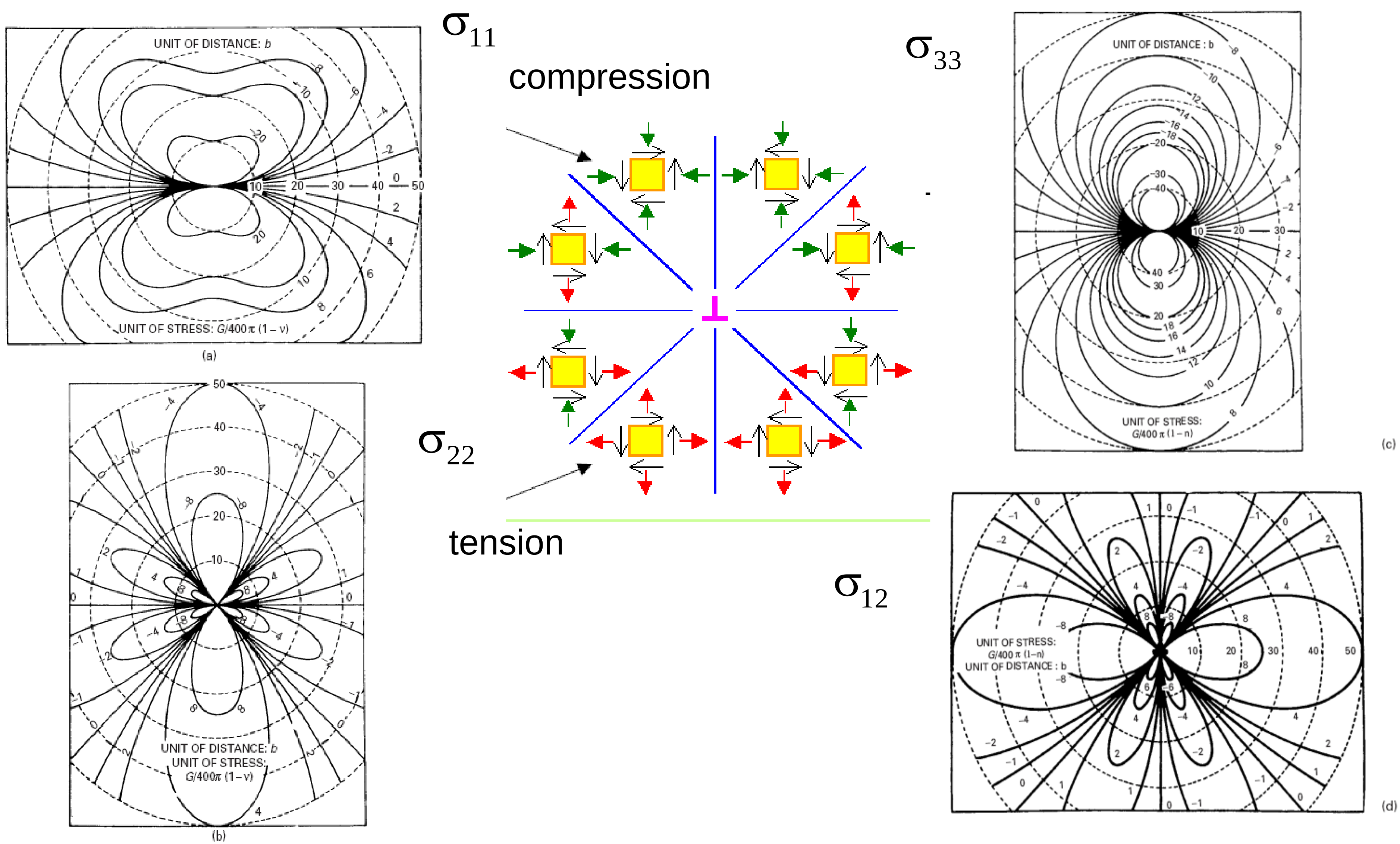
$$\sigma_{xx} = -\frac{Gb}{2\pi(1-\upsilon)}\frac{\sin\theta\left(2+\cos 2\theta\right)}{r}$$

$$\sigma_{yy} = \frac{Gb}{2\pi(1-\upsilon)}\frac{\sin\theta\cos 2\theta}{r}$$

$$\sigma_{zz} = \upsilon(\sigma_{xx} + \sigma_{yy})$$

$$\sigma_{xy} = \frac{Gb}{2\pi(1-\upsilon)}\frac{\cos\theta\cos 2\theta}{r}$$

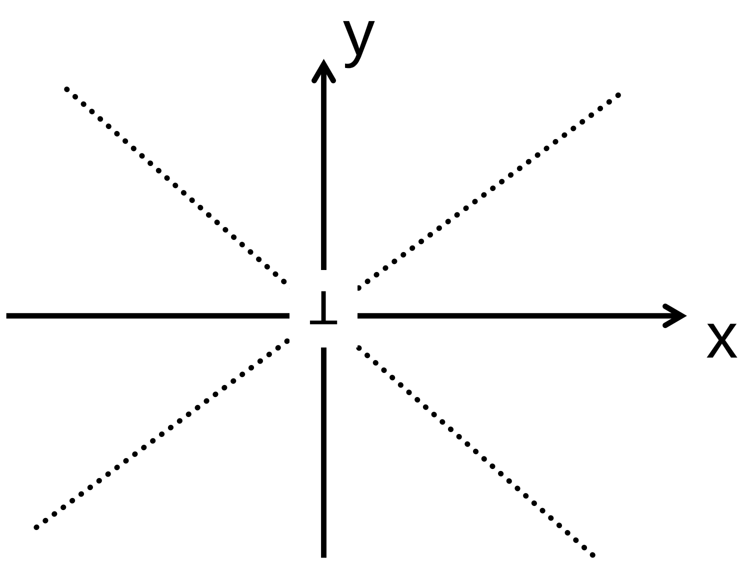
Stress Fields Around a Edge Dislocation



Forces on dislocations: Peach-Koehler equation

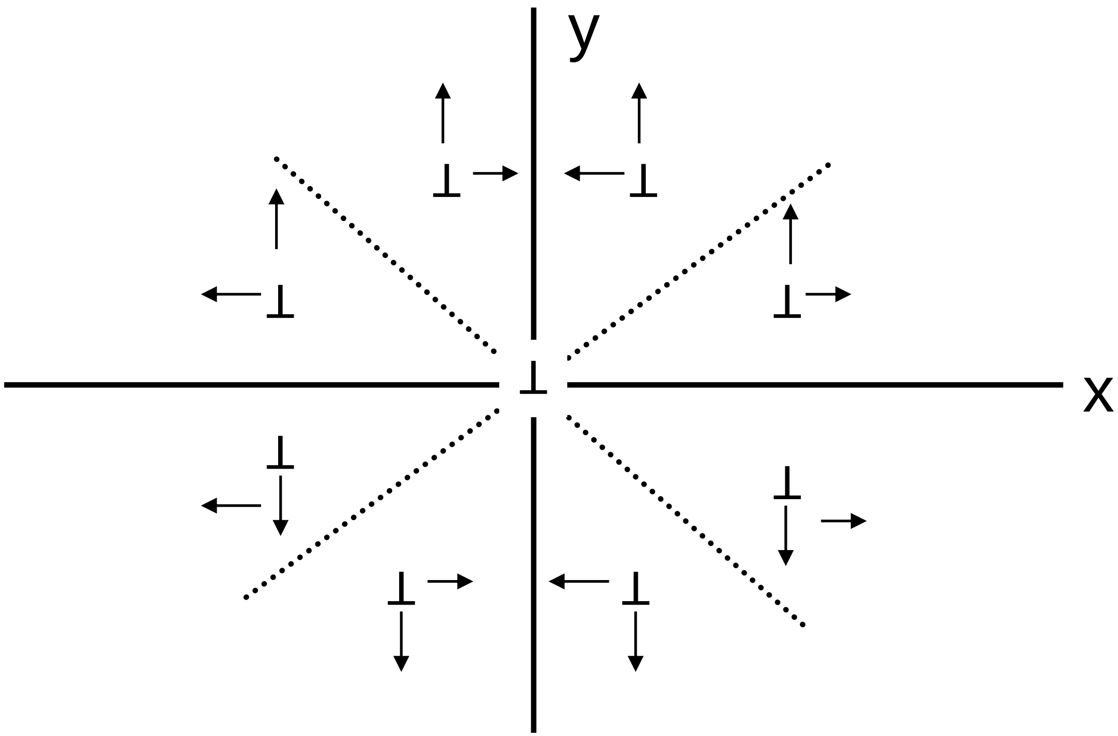
$$\vec{F}_{12} = (\sigma_1 \cdot \vec{b}_2) \times \vec{l}_2$$

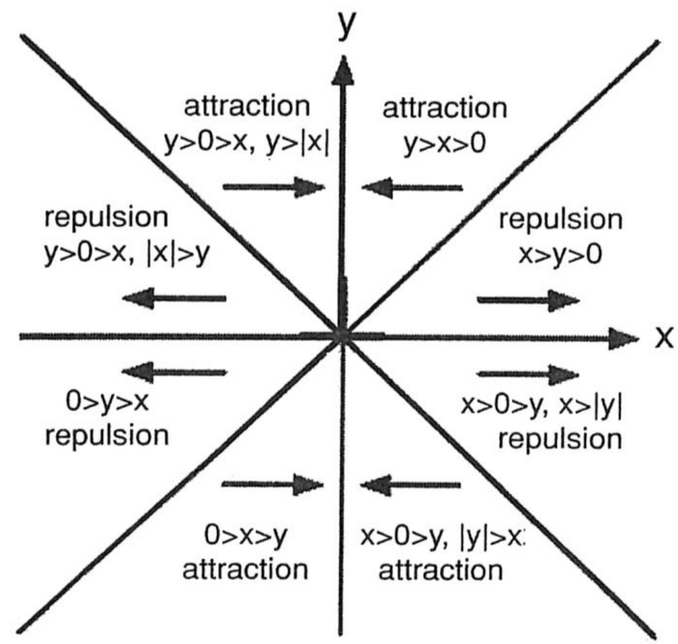
$$\sigma_1 = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \qquad \vec{b}_2 = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} \qquad \vec{l}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



$$F = \tau b$$

Forces on dislocations: Peach-Koehler equation



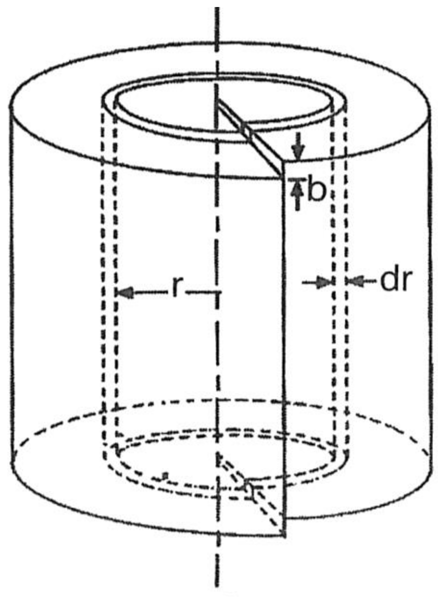


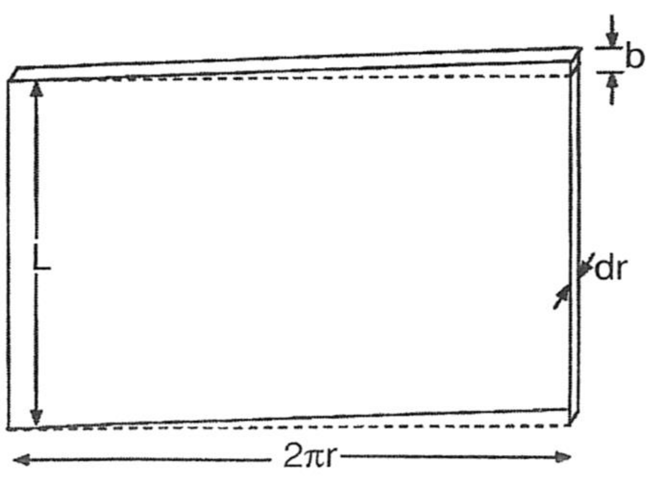
$$\vec{F}_{12} = \begin{pmatrix} \sigma_{xy} b_2 \\ -\sigma_{xx} b_2 \\ 0 \end{pmatrix}$$

$$\sigma_{xy} = \frac{G b_1 b_2}{2\pi(1-\upsilon)} \frac{x(x^2 - y^2)}{(x^2 + y^2)^2}$$

$$\sigma_{xx} = -\frac{G b_1 b_2}{2\pi(1-\upsilon)} \frac{y(3x^2 + y^2)}{(3x^2 + y^2)^2}$$

Energy of screw dislocation





$$\tau_{\theta z} = G\gamma \qquad \gamma = \frac{b}{2\pi r}$$

$$E_v = \frac{1}{2}\tau_{\theta z}\gamma$$

$$E_v = \frac{G\gamma^2}{2}$$

$$E_v = \frac{Gb^2}{8\pi^2 r^2}$$

Energy of screw dislocation

$$E_{\text{screw}} = \int_0^{2\pi}\int_{r_0}^{r} \frac{Gb^2}{8\pi^2 r^2}\cdot r\,dr\,d\theta \cdot l$$

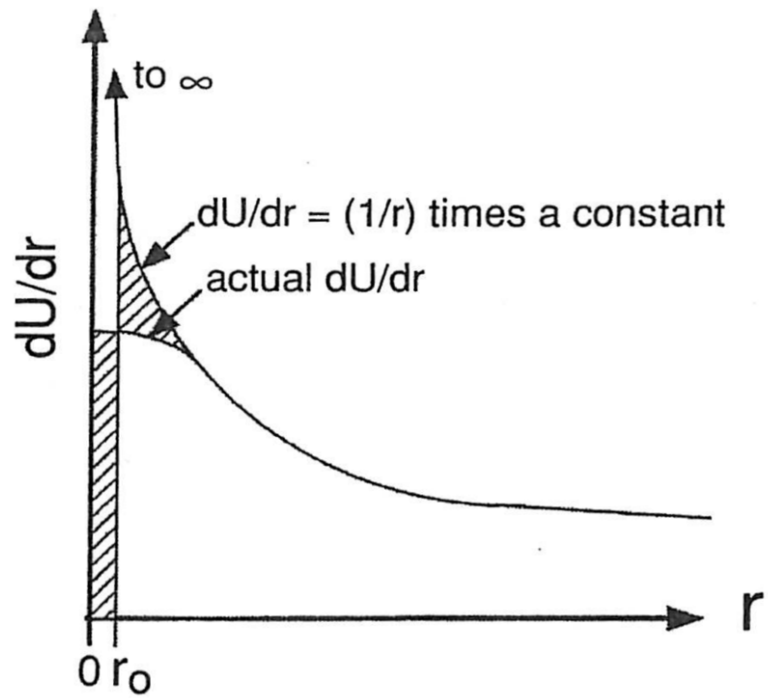
$$= l \cdot \frac{Gb^2}{4\pi}\ln\left(\frac{r}{r_0}\right)$$

$$\frac{E_{screw}}{l} = \frac{Gb^2}{4\pi}\ln\left(\frac{r}{r_0}\right)$$

$$\frac{E_{edge}}{l} = \frac{Gb^2}{4\pi(1-\nu)}\ln\left(\frac{r}{r_0}\right)$$

$$\frac{E_{\text{dis}}}{l} \approx \frac{1}{2}Gb^2$$

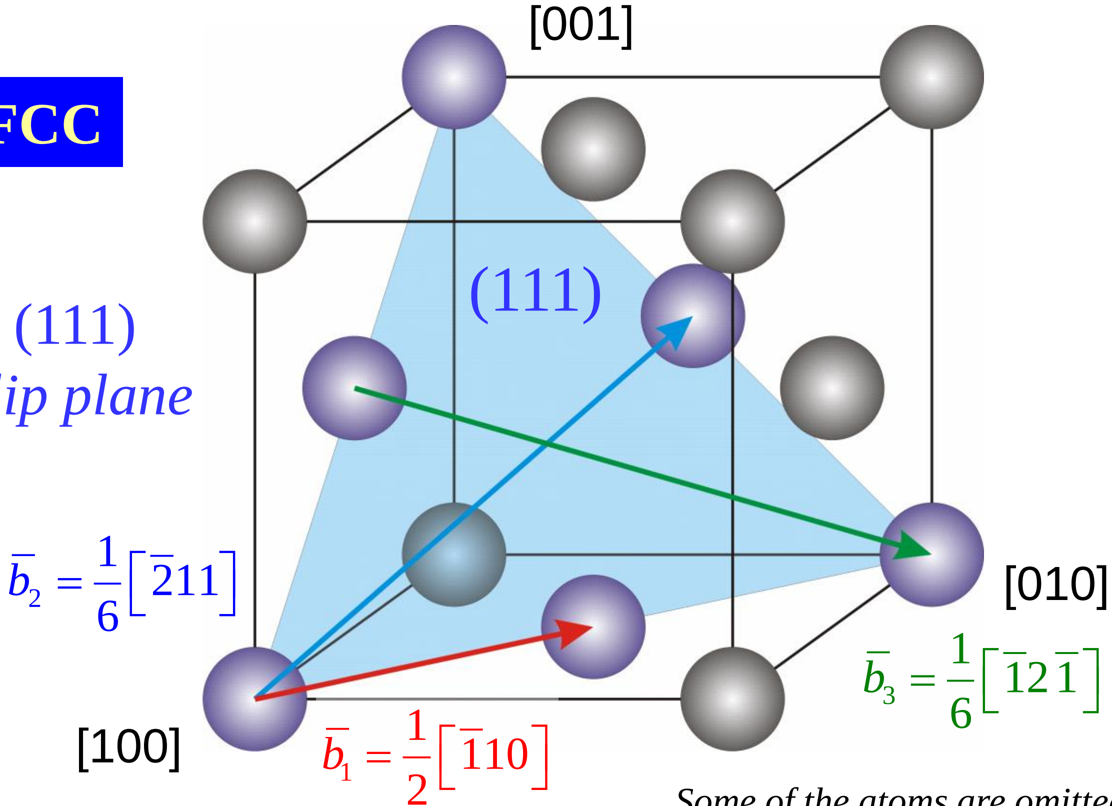
The strain energy per unit length in order to obtain minimum energy, that is, to posse a line tension analogous the surface tension.

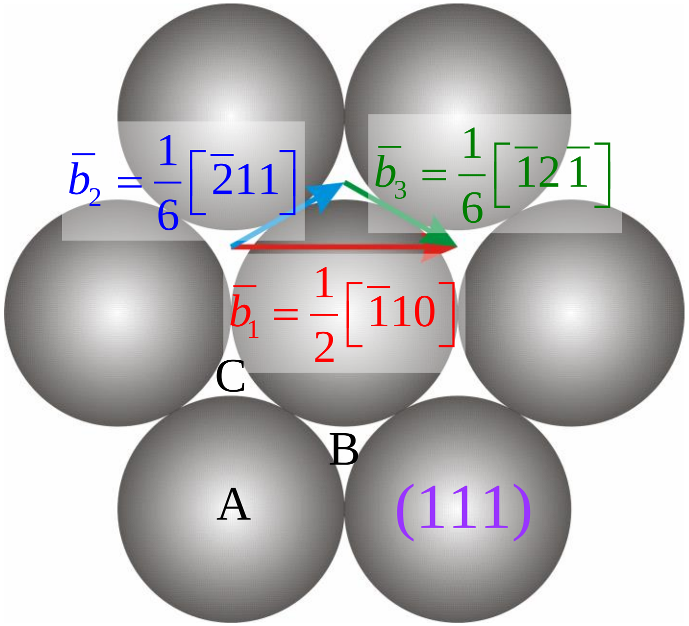


Partial dislocations in fcc

FCC

(111)
Slip plane

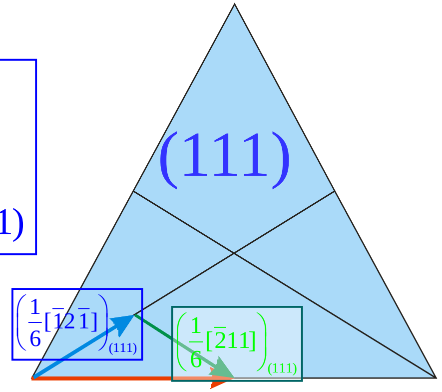




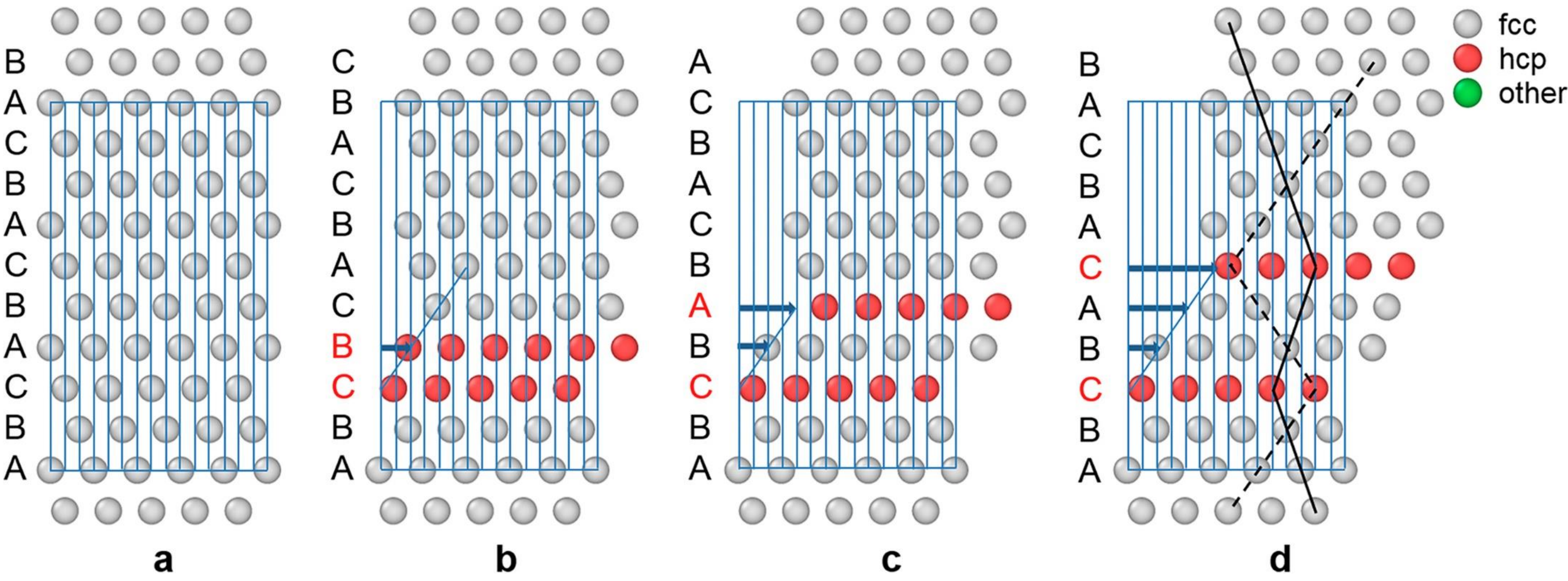
Some of the atoms are omitted for clarity

$$\left(\frac{1}{2}[\bar{1}10]\right)_{(111)} \rightarrow \left(\frac{1}{6}[\bar{1}2\bar{1}]\right)_{(111)} + \left(\frac{1}{6}[\bar{2}11]\right)_{(111)}$$

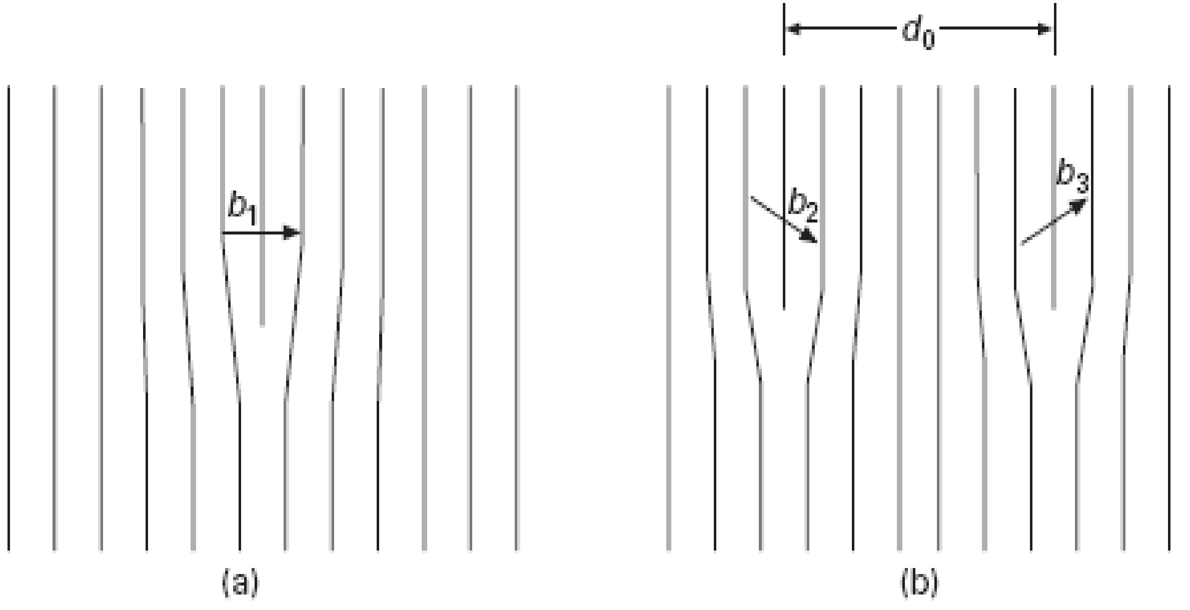
Shockley Partials



Dislocations and Stacking faults



Partial dislocations



$$U_L \approx Gb^2$$

$$U_L = \tfrac{1}{2}Gb_1^2 \quad > \quad U_L = \tfrac{1}{2}\left(Gb_2^2 + Gb_3^2\right)$$

Stacking faults in fcc

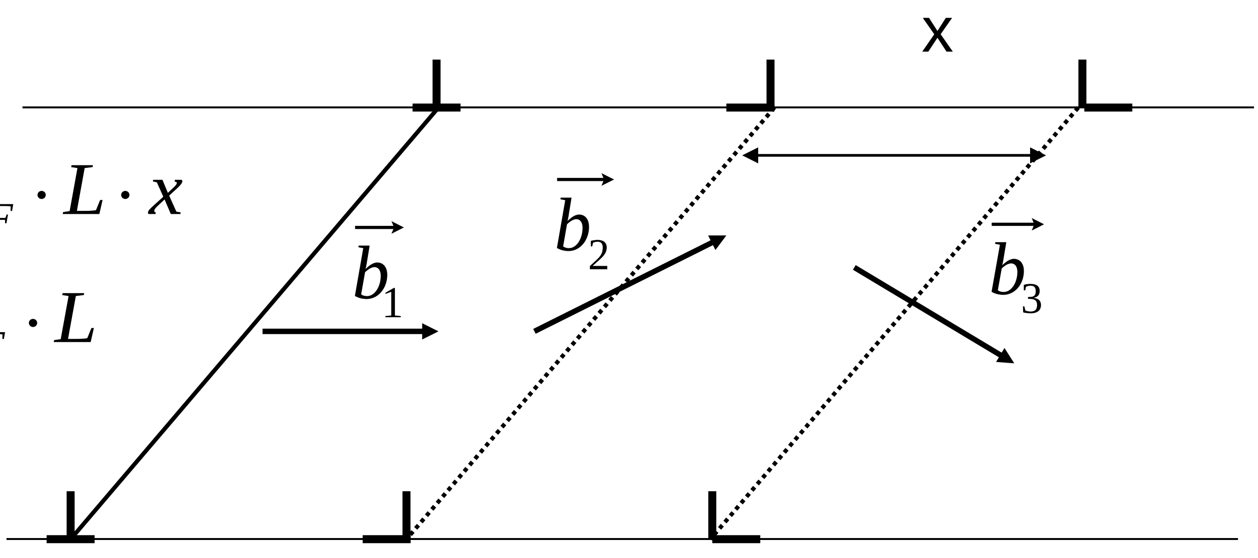
Table 9.2. Stacking fault energies of several fcc metals

Ag	Al	Au	Cu	Ni	Pd	Pt	Rh	Ir
16	166	32	45	125	180	322	750	300 mJ/m^2.

Source From listing in J. P. Hirth and J. Lothe, *Theory of Dislocations*, Wiley, (1982).

$$E_{SF} = \gamma_{SF} \cdot L \cdot x$$

$$F_{SF} = \gamma_{SF} \cdot L$$



Stacking faults energy

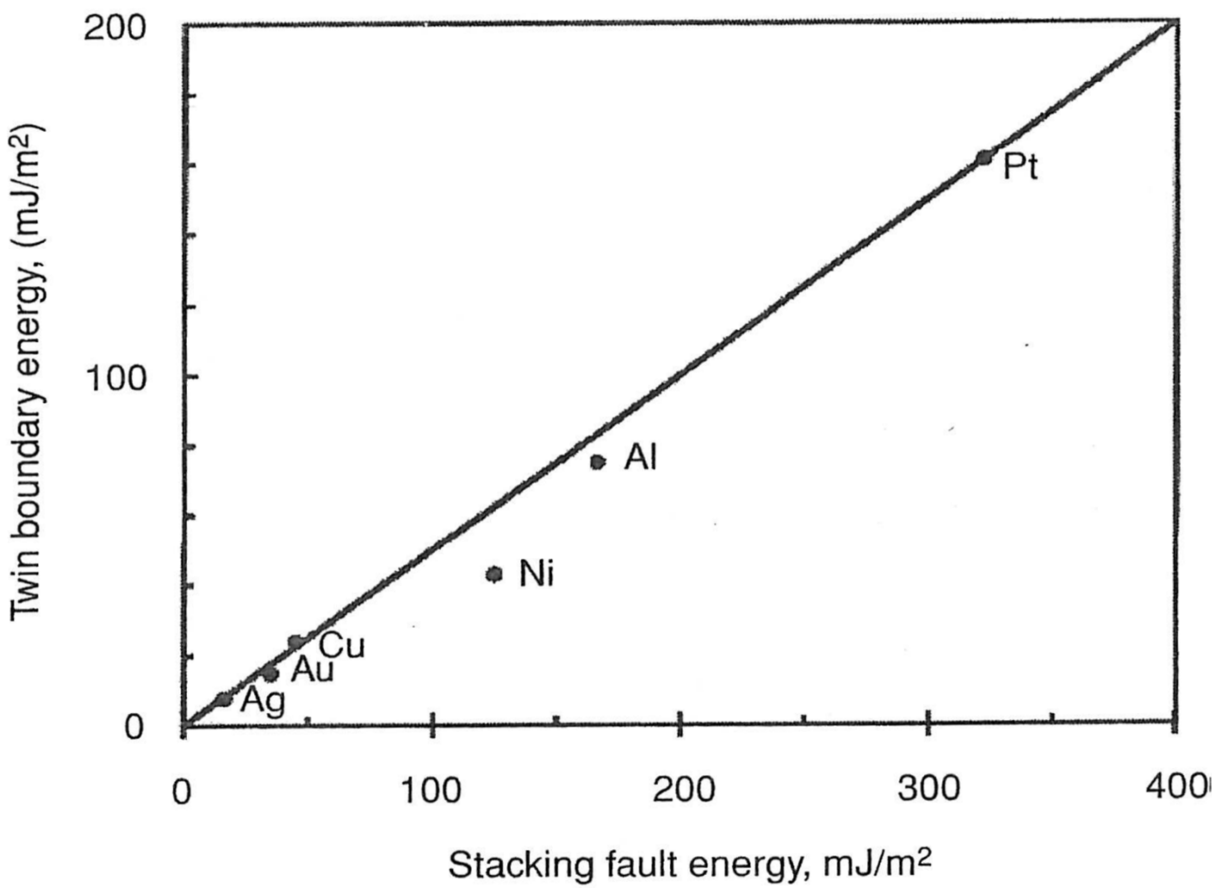


Figure 9.24. The relationship of stacking fault energy and twin boundary energy. The line represents $\gamma_{\text{stacking fault}} = 2\gamma_{\text{twin}}$.

References:

1. A. Kelly, G.W. Groves and P. Kidd, “Crystallography and crystal defects”, revised edition, 2000, John Wiley & Sons Ltd., England.
2. G. Gottstein, “Physical Foundations of Materials Science”, 2004, Springer Verlag, Heidelberg.