

# Mechanical Behaviour of Materials

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## Chapter 06 Strengthening

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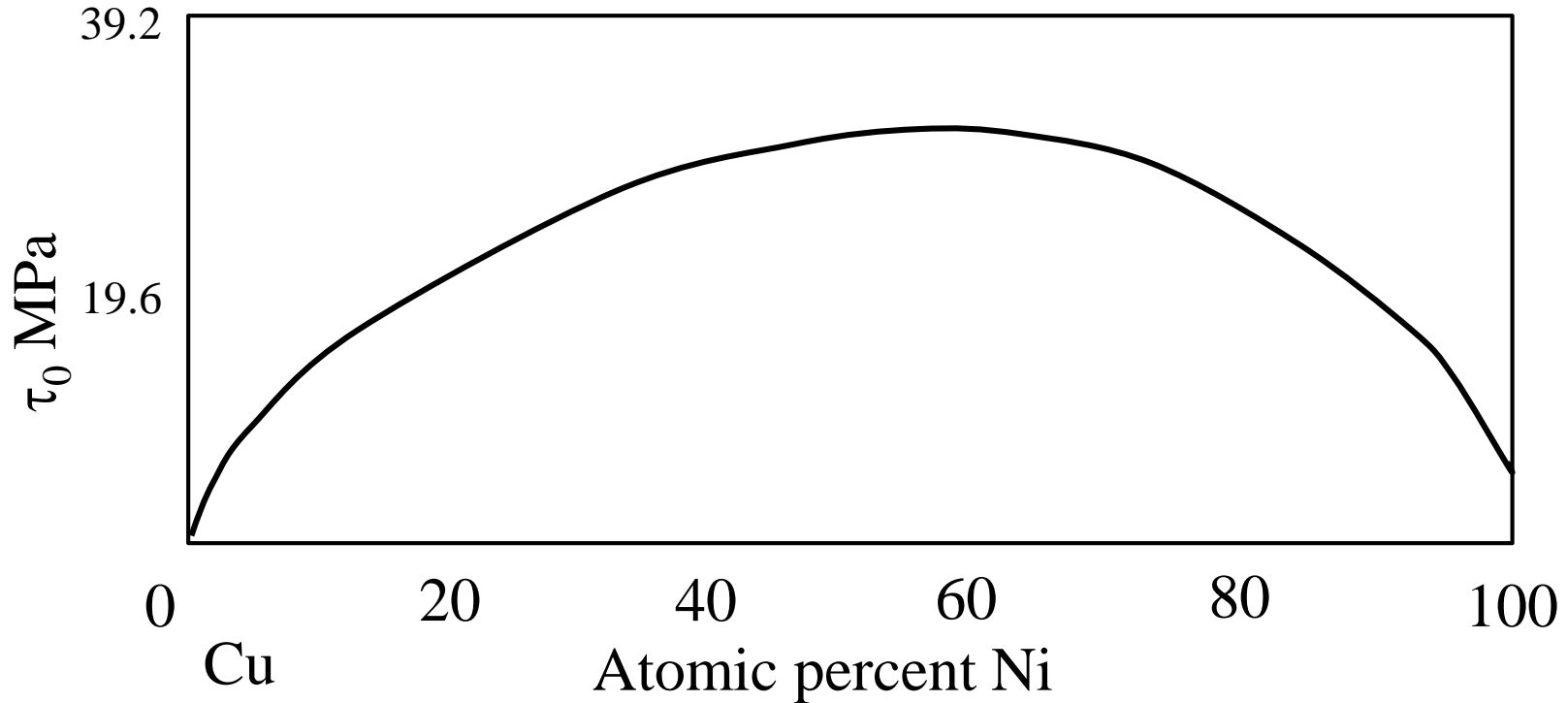
Dr.-Ing. 郭瑞昭

# Introduction to Alloy Strengthening

- Hardening by solute atoms
  - Solid solution strengthening  
(interaction between solute atoms and dislocations)
  
- Hardening by second phases
  - Precipitation-Hardening  
(interaction between precipitation phases and dislocations)
  - Dispersion-Hardening  
(interaction between dispersion phases and dislocations)

Solid solution strengthening:  
0 dimension defects

# Effect of alloying on strengthening a material



Critical shear stress of Cu-Ni solid solution crystals:

- a) Formation of an internal stress due to the size and modulus effect and b) interaction with dislocations

# Solid solution strengthening

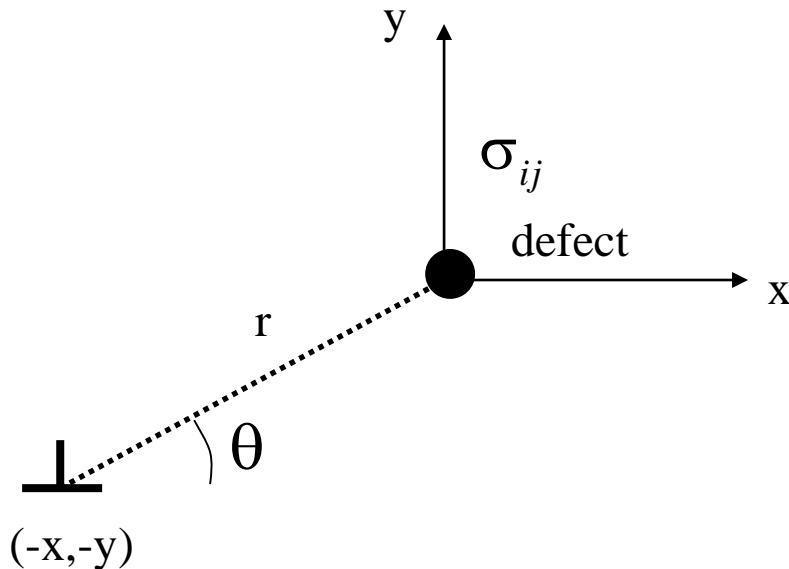
In addition to elastic misfit interaction, there are other sources of dislocation-solute interactions:

1. Parelatic interaction (lattice parameter effect)
2. Dielastic interaction (shear modulus effect)
3. Chemical interaction (Suzuki effect)

However, their contributions are less important than the size effect.

# Elastostatic interaction (Lattice parameter effect)

One of the simplest to understand is the interaction between the elastic stress fields of a dislocation and a point defect. Cottrell was the first to calculate the energy involved in such an interplay fields.



The only stresses acting on edge dislocation are

$$\sigma_{xy} \quad \sigma_{xx}$$

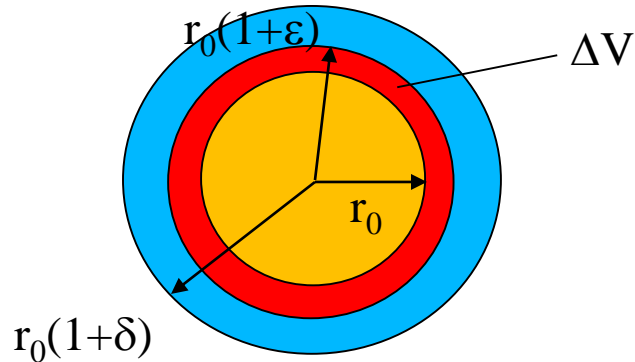
$$\vec{F}_{def-dis} = (\sigma_{def} \cdot \vec{b}_{dis}) \times \vec{l}_{dis}$$

$$\vec{F}_{def-dis} = \begin{pmatrix} \sigma_{xy} b \\ -\sigma_{xx} b \\ 0 \end{pmatrix}$$

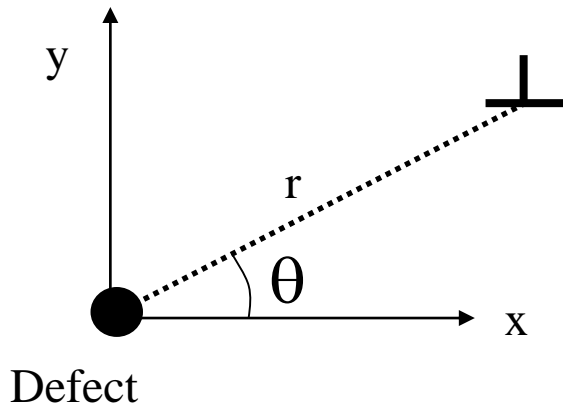
# Elastic Interactions

Misfit volume:  $\Delta V$

Distortion induced by point defects



- Radius of a hole :  $r_0$
- Nature radius of defect:  $r_0(1+\delta)$
- Final radius of defect:  $r_0(1+\varepsilon)$



$$\Delta V_e = \left( \frac{4}{3} \right) \pi r_0^3 (1 + \varepsilon)^3 - \frac{4}{3} \pi r_0^3$$

$$= \frac{4}{3} \pi r_0^3 [(1 + \varepsilon)^3 - 1]$$

$$\approx \frac{4}{3} \pi r_0^3 [1 + 3\varepsilon - 1]$$

$$\Delta V_e \approx 4\pi r_0^3 \varepsilon$$

$$V_{mis} = V_s - V_e = 4\pi r_0^3 \delta$$

$$\Delta V_e = \frac{(1 + \nu)}{3(1 - \nu)} V_{mis}$$

$$\Delta V = \frac{3(1 - \nu)}{(1 + \nu)} \Delta V_e = V_{mis}$$

Total volume change

## Elastic Interactions-conti.

$$E^P = \sigma_p \cdot \Delta V \cdot \left( 3 \frac{1-\nu}{1+\nu} \right)$$

$$\sigma_p = \frac{1}{3} (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

$$= \frac{1+\nu}{1-\nu} \frac{Gb \sin \theta}{3\pi r}$$

$$\frac{d}{dx} \left[ \frac{x}{x^2 + y^2} \right] =$$

$$\frac{1}{x^2 + y^2} + x \frac{-2x}{(x^2 + y^2)^2}$$

$$F^P = - \frac{dE^P}{dx}$$

$$= - \frac{d}{dx} \left[ \left( - \frac{Gb}{3\pi} \frac{1+\nu}{1-\nu} \frac{x}{x^2 + y^2} \right) \Delta V \left( 3 \frac{1+\nu}{1-\nu} \right) \right]$$

$$= \frac{d}{dx} \left[ \frac{Gb}{\pi} \frac{x}{x^2 + y^2} \Delta V \right]$$

$$= \frac{Gb}{\pi} \Delta V \frac{d}{dx} \left[ \frac{x}{x^2 + y^2} \right]$$

$$= \frac{Gb}{\pi} \Delta V \left[ \frac{y^2 - x^2}{(x^2 + y^2)^2} \right]$$



## Elastic Interactions-conti.

$$F^P = \frac{Gb}{\pi} \Delta V \left[ \frac{y^2 - x^2}{(x^2 + y^2)^2} \right]$$

$$\Delta V = (a(1 + \delta))^3 - a^3$$

$$\delta = \frac{da}{dc} \frac{1}{a} = \frac{d \ln a}{dc}$$

$\delta$  is a factor of lattice change  
and  $c$  the concentration

$F^P$  is maximum, when  $x = \frac{y}{\sqrt{3}}$

$$y = \frac{b}{\sqrt{3}} \quad x = \frac{b}{3}$$

$$F_{\max}^P = \frac{9}{8} \frac{G}{b\pi} \Delta V$$

$$= \frac{9}{8} \frac{G}{b\pi} [a^3(1 + 3\delta + 3\delta^2 + \delta^3) - a^3]$$

$$= \frac{9}{8} \frac{G}{b\pi} [a^3(1 + 3\delta + 3\delta^2 + \delta^3) - a^3]$$

$$= \frac{9}{8} \frac{G}{b\pi} 3\delta a^3 \approx Gb^2 \delta$$

$\delta$  is small and  $b$  is equal to  $a$

The interaction force of  
substitutional atoms leads to a  
linear dependence on the lattice  
distortion.

## Dielastic Interactions (shear modulus effect)

$$E^d = \frac{Gb^2}{8\pi^2 r^2} \Omega \eta$$

$$F_{\max}^d \approx \frac{1}{20} Gb^2 |\eta|$$

$$\eta = \frac{dG}{dc} \frac{1}{G} = \frac{d \ln G}{dc}$$

# Critical shear stress of solid solution strengthening

The maximum possible interaction force  $F_{\max} = F_{\max}^d + F_{\max}^p$

The critical shear stress for solid solution strengthening according Peach-Koehler Eq.

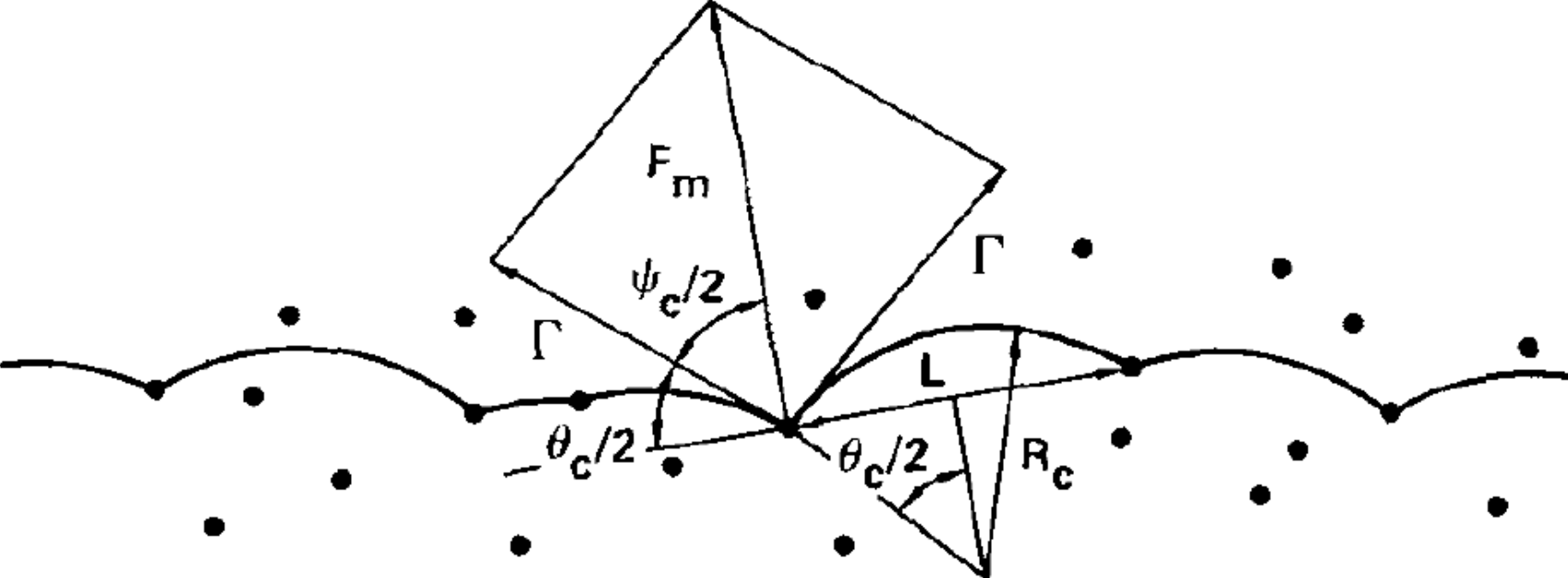
$$F_{\max} = \Delta\tau_c b l_F$$

The Friedel length  $l_F$  is the average obstacle spacing given by Friedel and  $\Lambda^2$  is the area of the glide plane per obstacle.

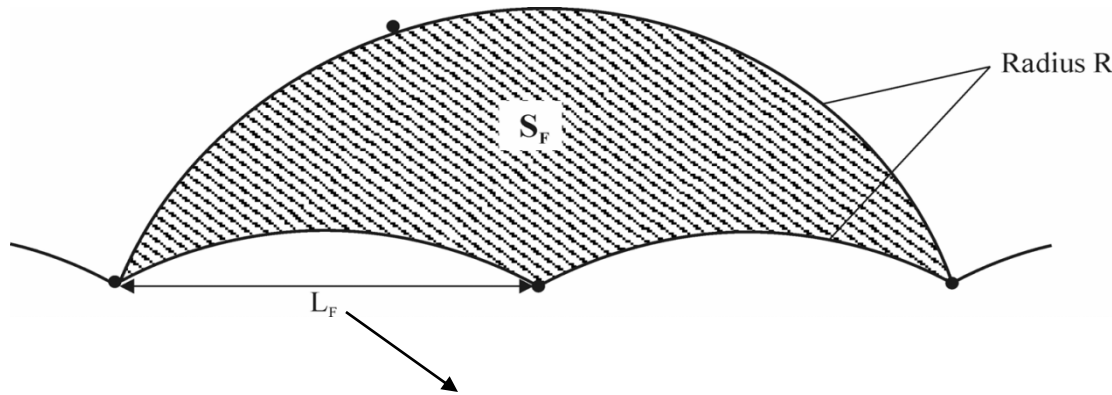
$$\Lambda^2 = \frac{\Delta\tau_c \cdot l_F^3}{Gb} = \frac{1}{c_F} \quad l_F = \left( \frac{Gb}{\Delta\tau_c \cdot c_F} \right)^3$$

If we consider the obstacle to be solute atoms with a concentration  $c_F$  atoms/m<sup>2</sup>.

# Schematic illustration of the penetration of a random array of point obstacles by a dislocation



# Solid Solution Strengthening: Elastic Interactions



average free dislocation length

$$F_{\max} = \Delta\tau_c b l_F$$

$$\Delta\tau_c = \frac{F_{\max}^{3/2} \sqrt{c_F}}{b\sqrt{G}} \approx \sqrt{c_F}$$

$$l_F = \left( \frac{Gb}{\Delta\tau_c \cdot c_F} \right)^{1/3}$$

Increase in the critical shear stress due to the solid solution strengthening is proportional to  $\sqrt{c_F}$ .

# Solid Solution Strengthening: Elastic Interactions

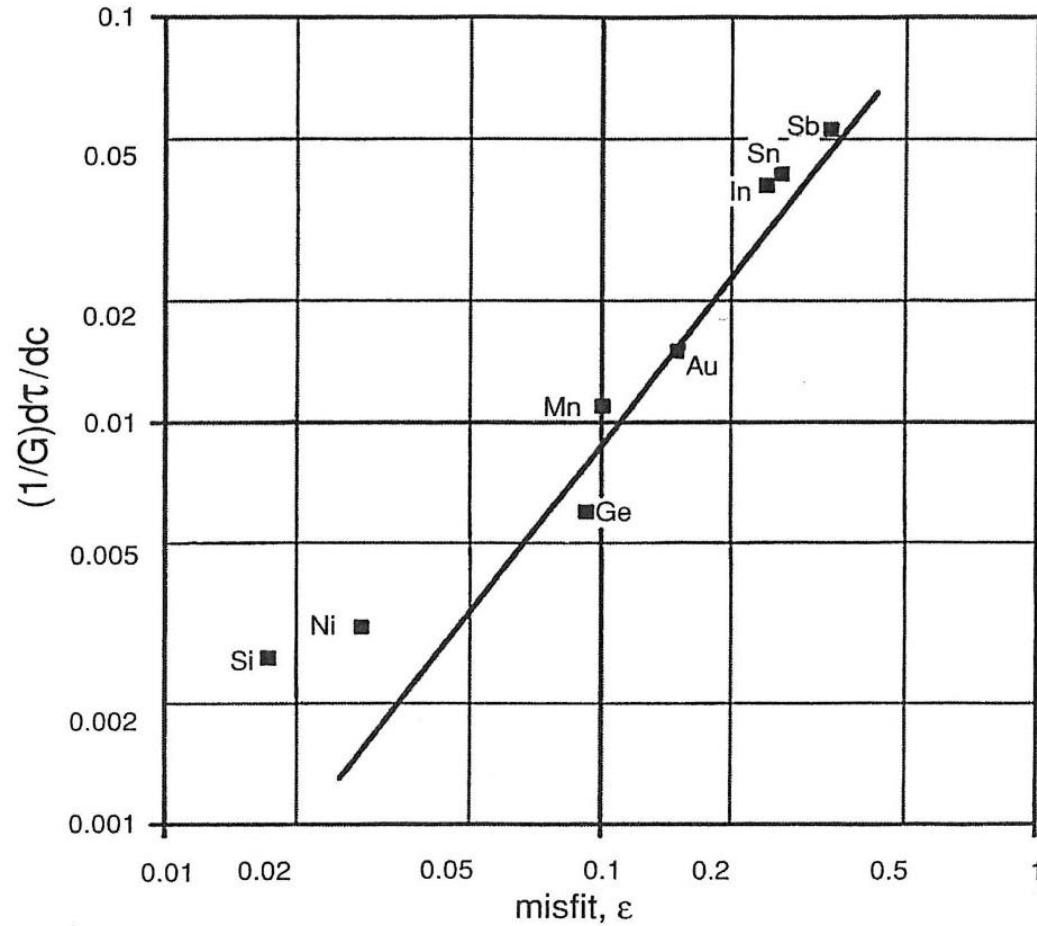
$$F_{\max} = F_{\max}^d + F_{\max}^p$$

$$\Delta\tau_c = \frac{(Gb^2)^{3/2} \left( |\delta| + \frac{1}{20} |\eta| \right)^{3/2} \sqrt{c_F}}{b\sqrt{G}}$$

$$\Delta\tau_c = Gb^2 \sqrt{c_F} \left( |\delta| + \frac{1}{20} |\eta| \right)^{3/2}$$

The slope of the curve  $\Delta\tau_c - \sqrt{c_F}$

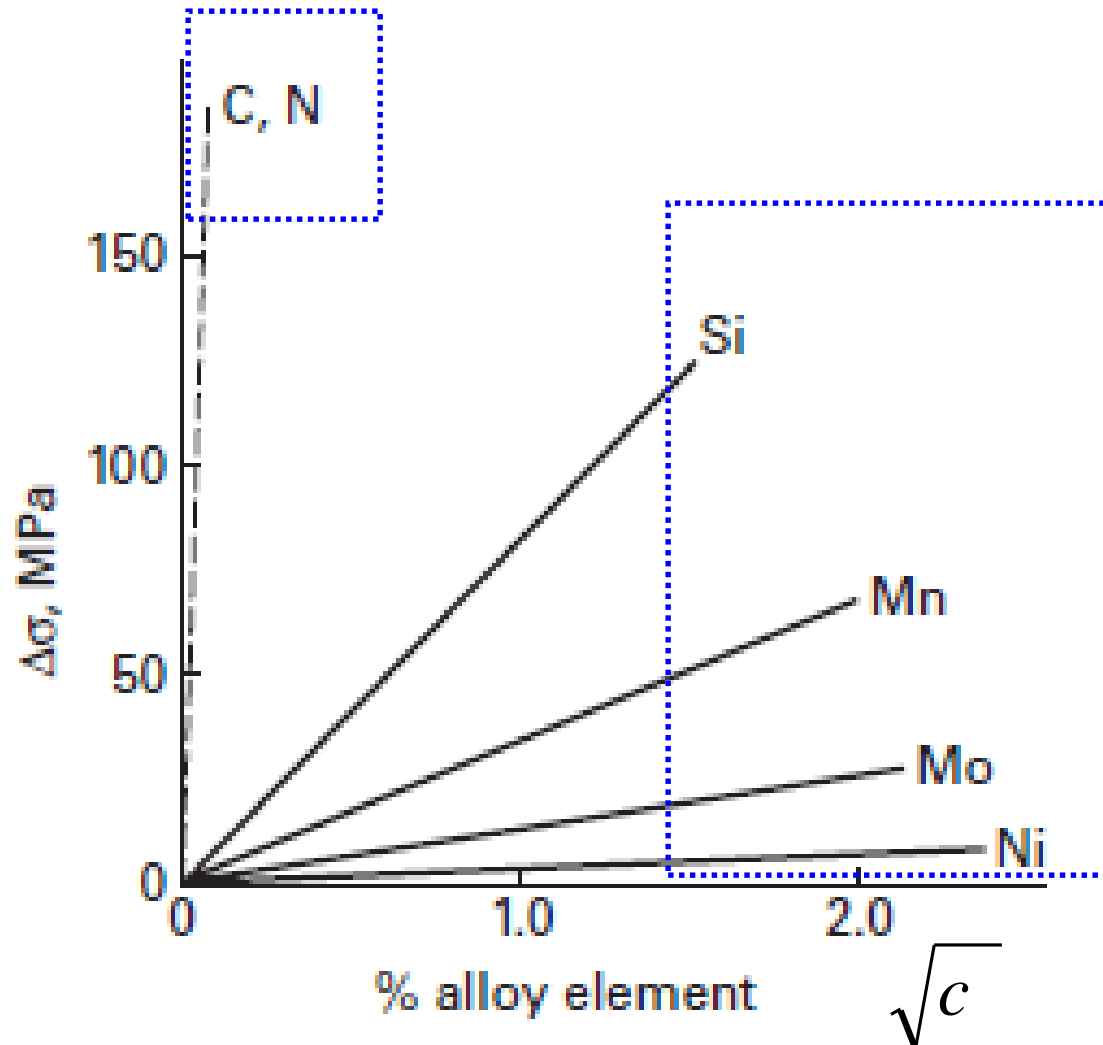
# Solid solution strengthening



$\delta =$

- 0.1 to 0 for vacancies
- 0.15 to +0.15 for substitutional solute
- 0.1 to 1.0 for interstitial solute

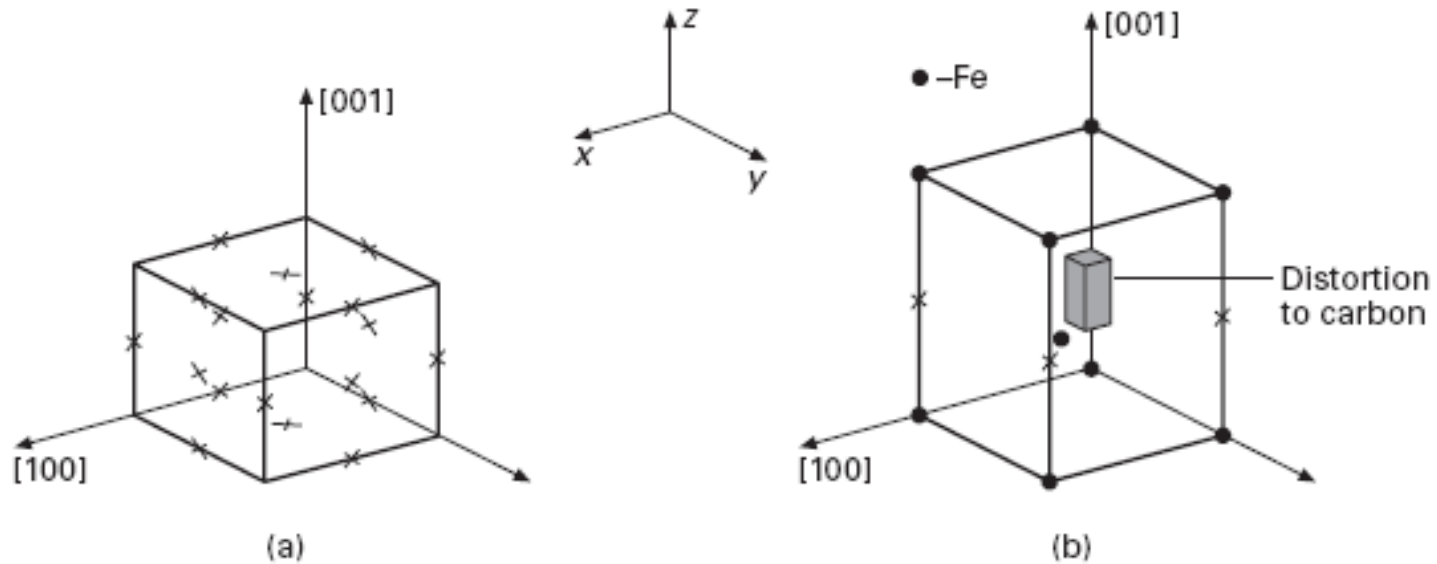
Why is the difference of Solid Solution Strengthening between substitutional and interstitial solution?



**Figure 12.10.** The effect of the solute misfit parameter,  $e = (1/a)da/dc$ , on the solute hardening of copper. The solid line is Equation (12.7) with  $C = 0.215$ . Data from F. McClintock and A. Argon, *ibid.*

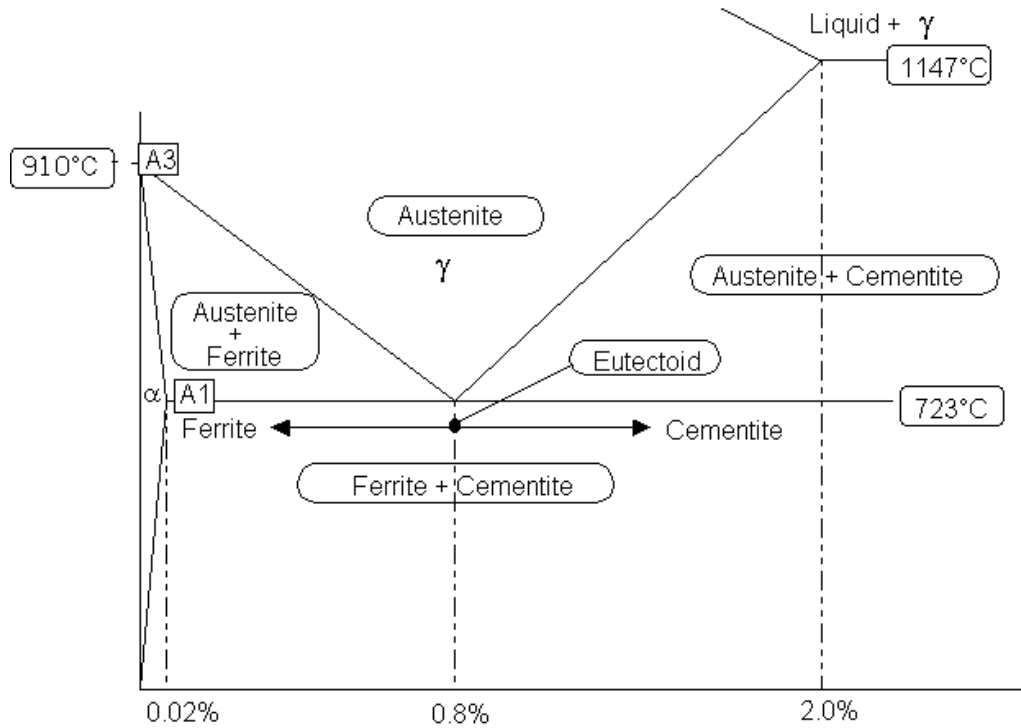


# Where are positions of Interstitial Atoms in the Cubic Lattice?



(a) Positions of interstitial atoms in the cube. (b) Carbon atom shown as a producer of a tetragonal distortion.

# Question?



$$a_C = 1.54 \text{ \AA}$$

$$a_{Fe} = 2.52 \text{ \AA}$$

$$R_C/R_{Fe} = 0.61$$

Octahedral

Tetrahedral

$$r/R_{fcc} = 0.41$$

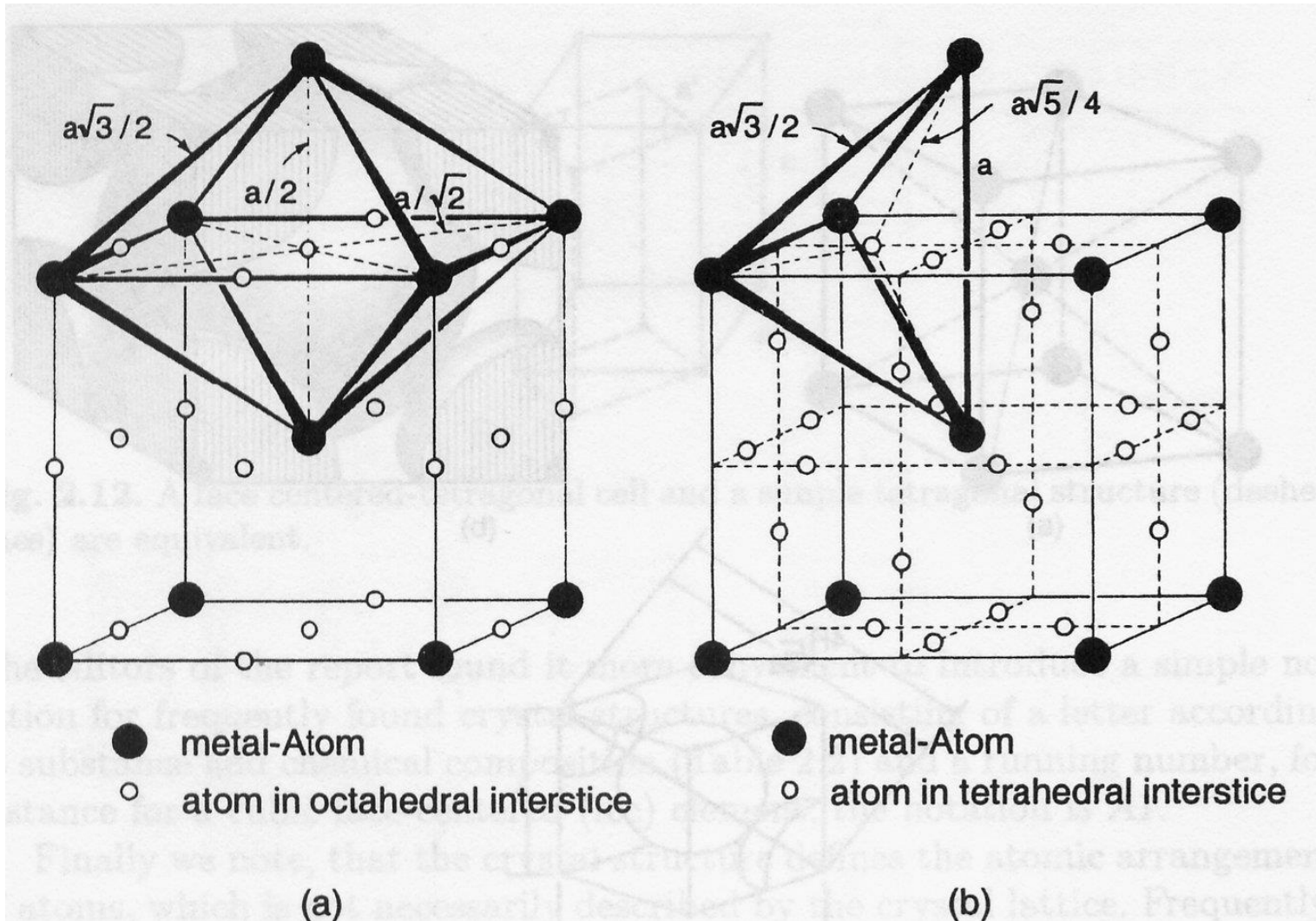
$$r/R_{fcc} = 0.22$$

$$r/R_{bcc} = 0.16$$

$$r/R_{bcc} = 0.29$$

# Bcc

$$R_C/R_{Fe} = 0.61$$

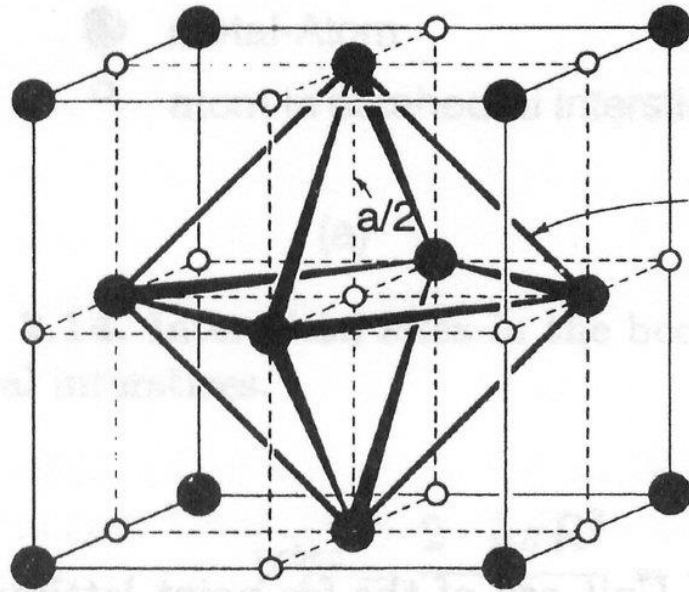


$$r/R = 0.155$$

$$r/R = 0.291$$

# Fcc

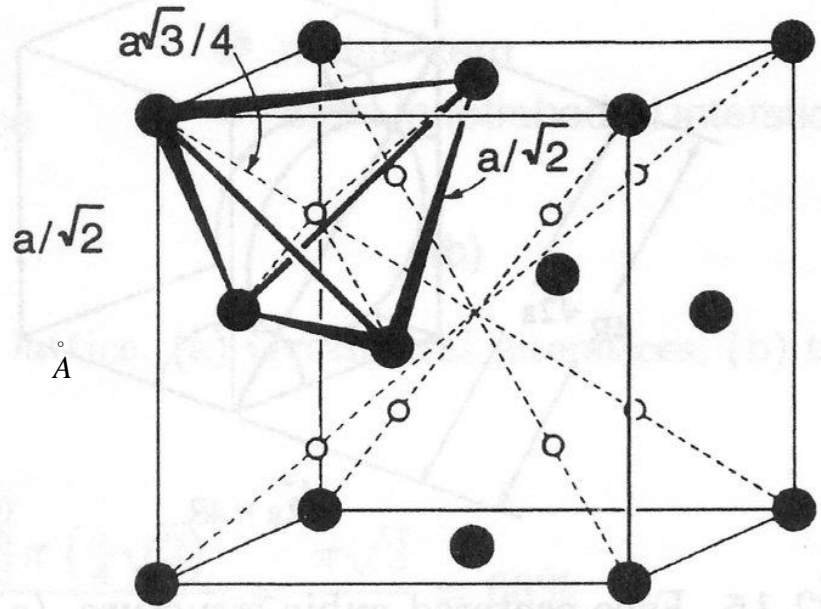
$$D_C = 1.54 \text{ \AA}$$
$$D_{Fe} = 2.52 \text{ \AA}$$



- metal atom
- atom in octahedral interstice

(a)

$$r/R = 0.41$$



- metal atom
- atom in tetrahedral interstice

(b)

$$r/R = 0.22$$

$$R_C/R_{Fe} = 0.61$$

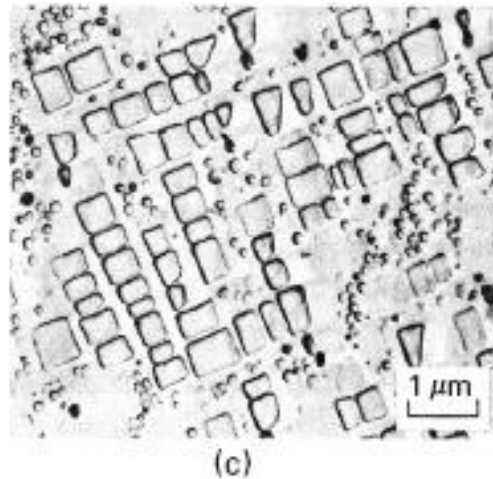
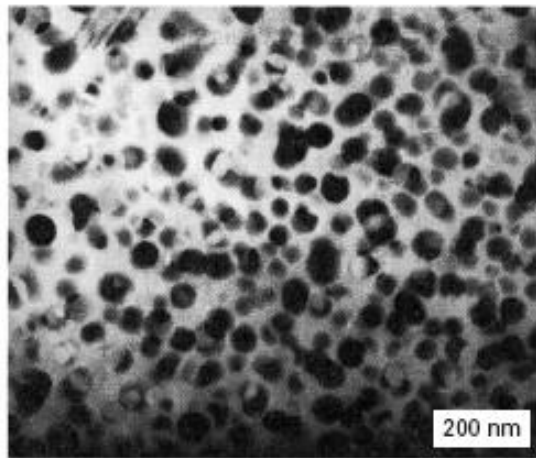
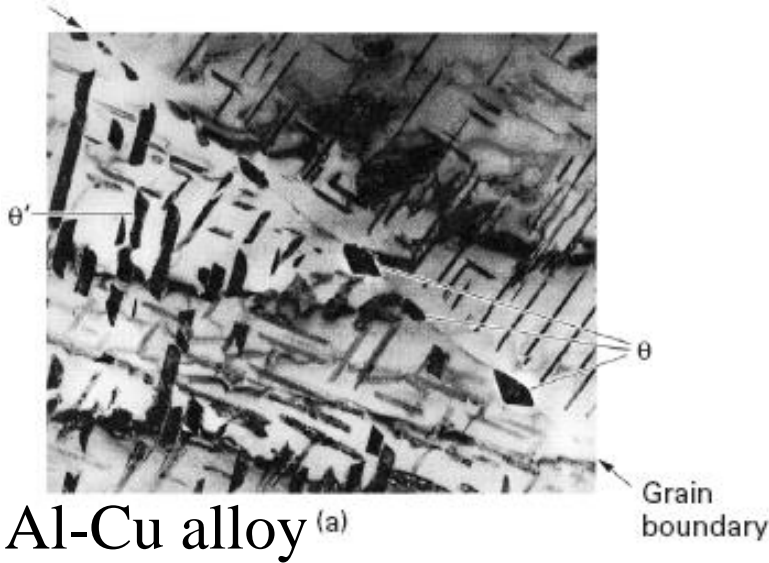
Strengthening mechanism:  
3 dimension defects

# Second Phases: Precipitation and Dispersion

Two alloy systems using this strengthening technique are aluminum alloys and nickel-based superalloys.

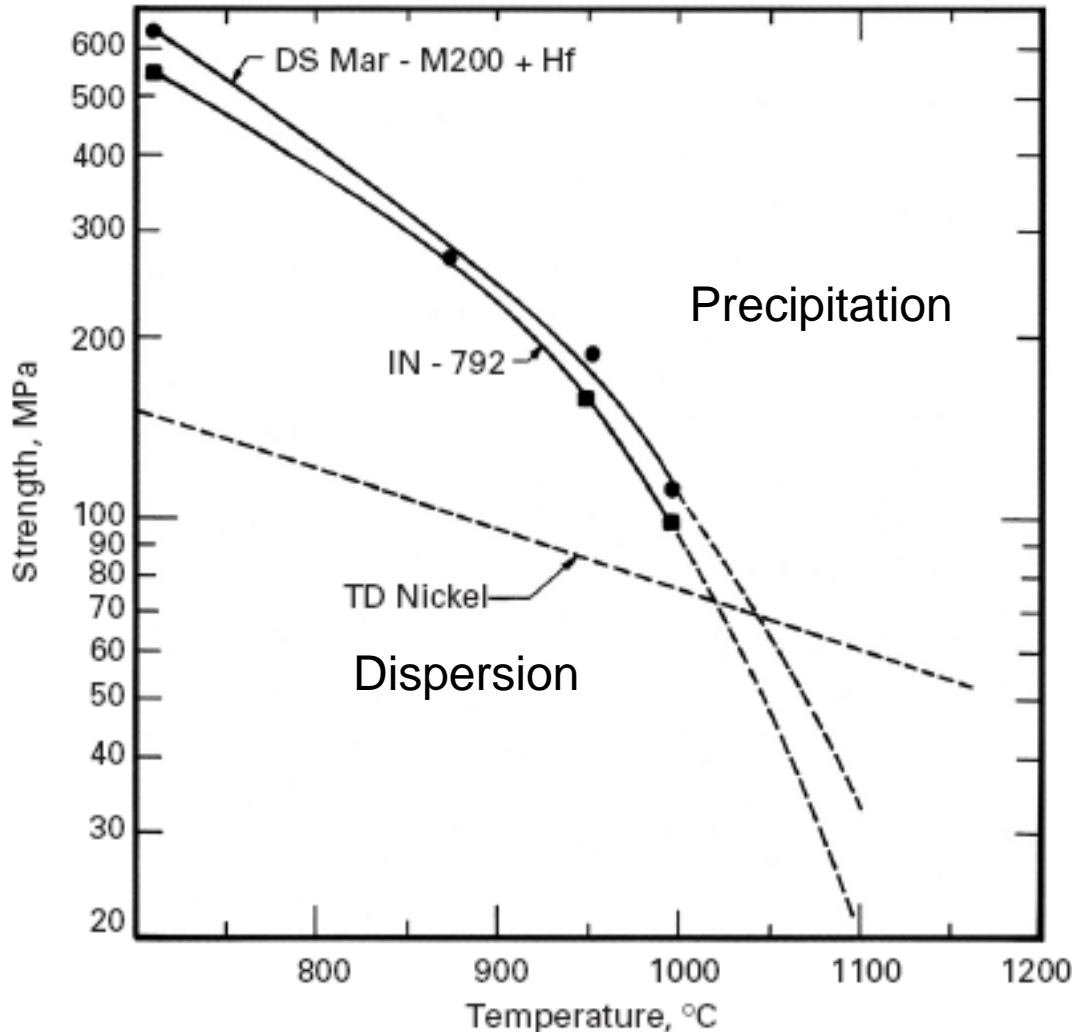
Precipitation: precipitation of precipitates out from a homogenous, supersaturated solid solution.

Dispersion: incorporated hard, insoluble second phases in a soft metallic matrix with a maximum volume fraction of 3-4%.



(a)  $\theta'$  precipitates (at grain boundaries) and  $\theta$  precipitates (in grain interior) in Al-Cu alloy. (Courtesy of K. S. Vecchio.) (b)  $\text{Al}_3\text{Li}$  precipitates in Al-Li alloy (TEM, dark field). (Courtesy of K. S. Vecchio.) (c)  $\gamma$  precipitates and aged carbides in a superalloy. (Courtesy of R. N. Orava.)

# Yield Strength Superalloys and TD Nickel



Comparison of yield strength of dispersion-hardened thoria-dispersed (TD) nickel with two nickel-based superalloys strengthened by precipitates (IN-792) and directionally solidified (DS) MAR M 200.

# Yield Strength of copper and copper alloys

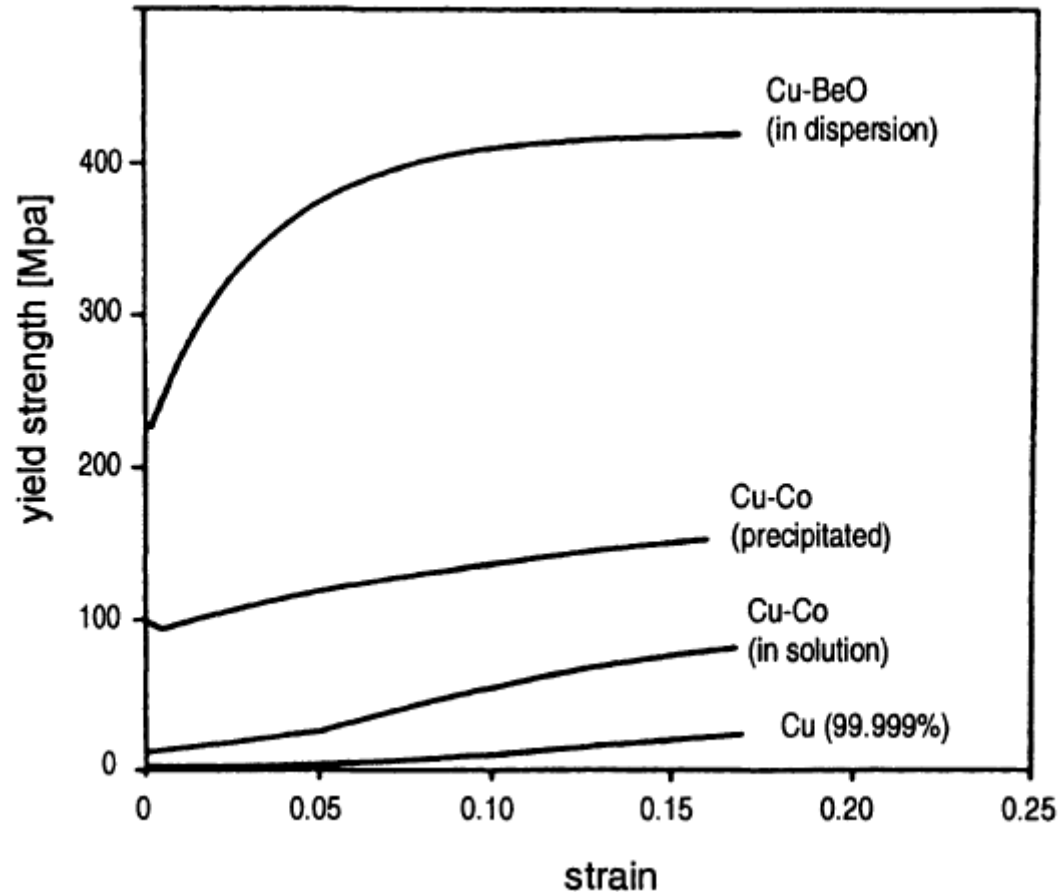


Fig. Hardening curves of high purity copper and copper alloys. In CuCo particles are shearable, in BeO they are not (after [6.24]).



# Dispersion strengthening: by-passing

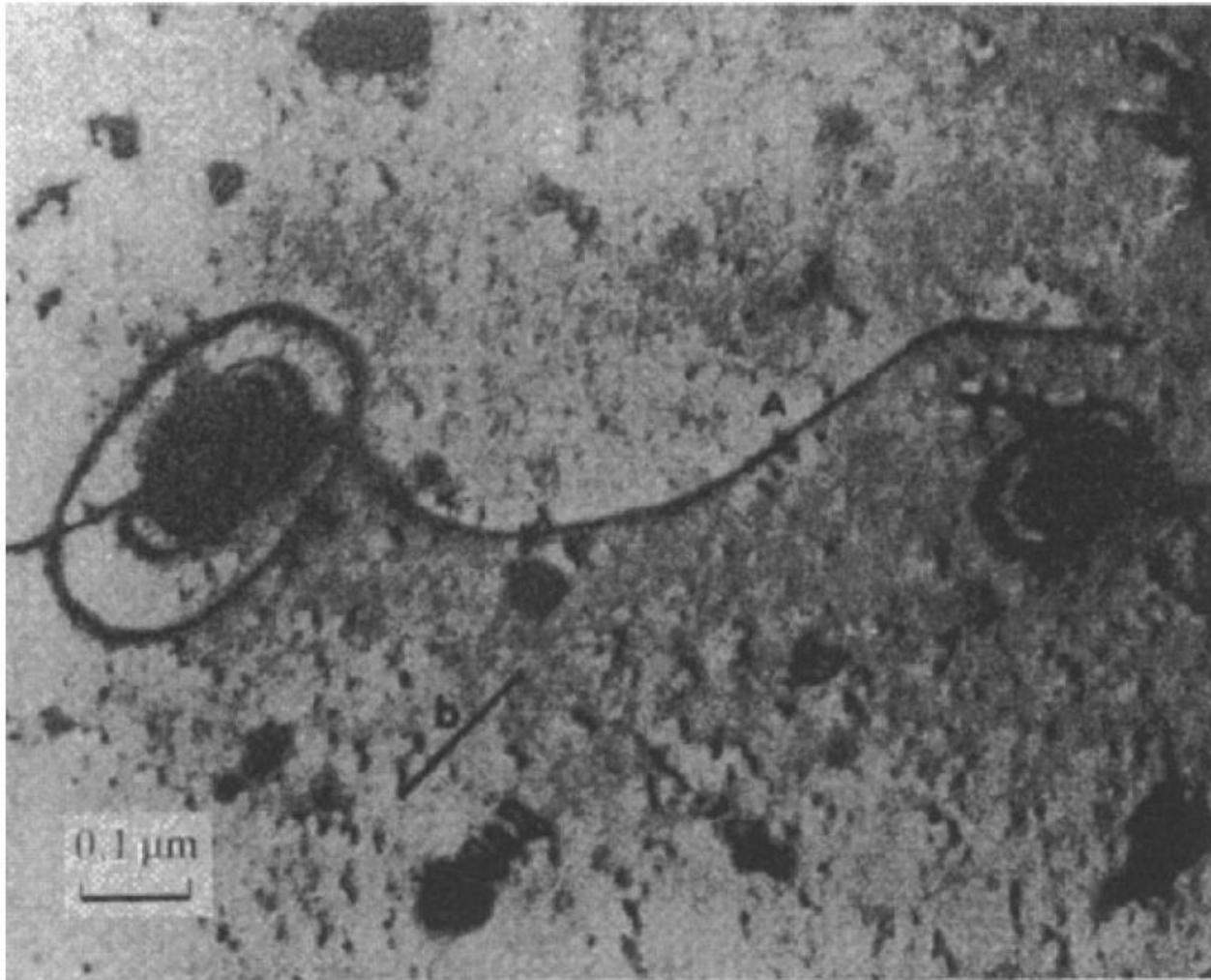
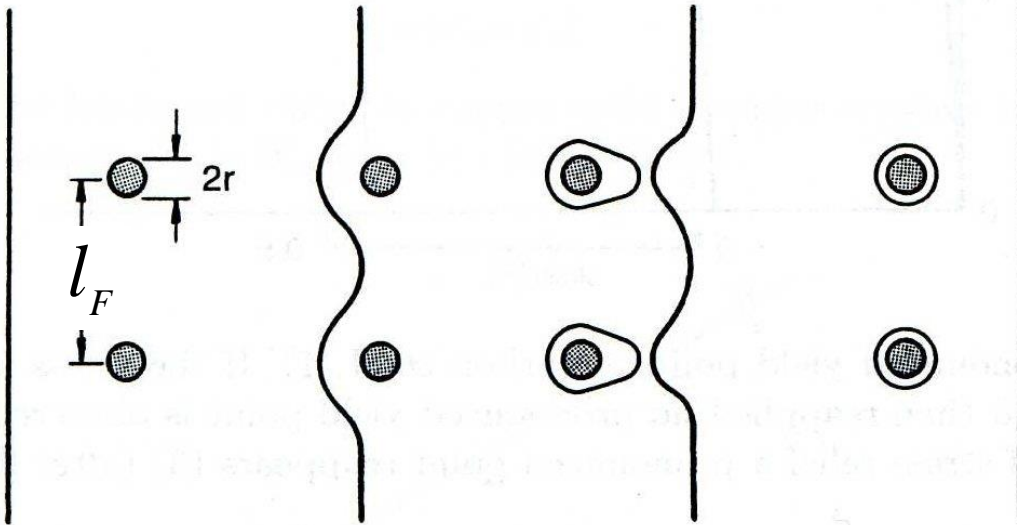


Fig. Orowan loops around  $\text{Al}_2\text{O}_3$  particles in  $\text{Cu}_{30\%}\text{Zn}$  [6.23].

# Dispersion strengthening

The only possible deformation mechanism is the Orowan passing stress:  
Dislocations circumnavigating around the particles in the slip plane



$$\Delta\tau_{OR} b l_F =$$

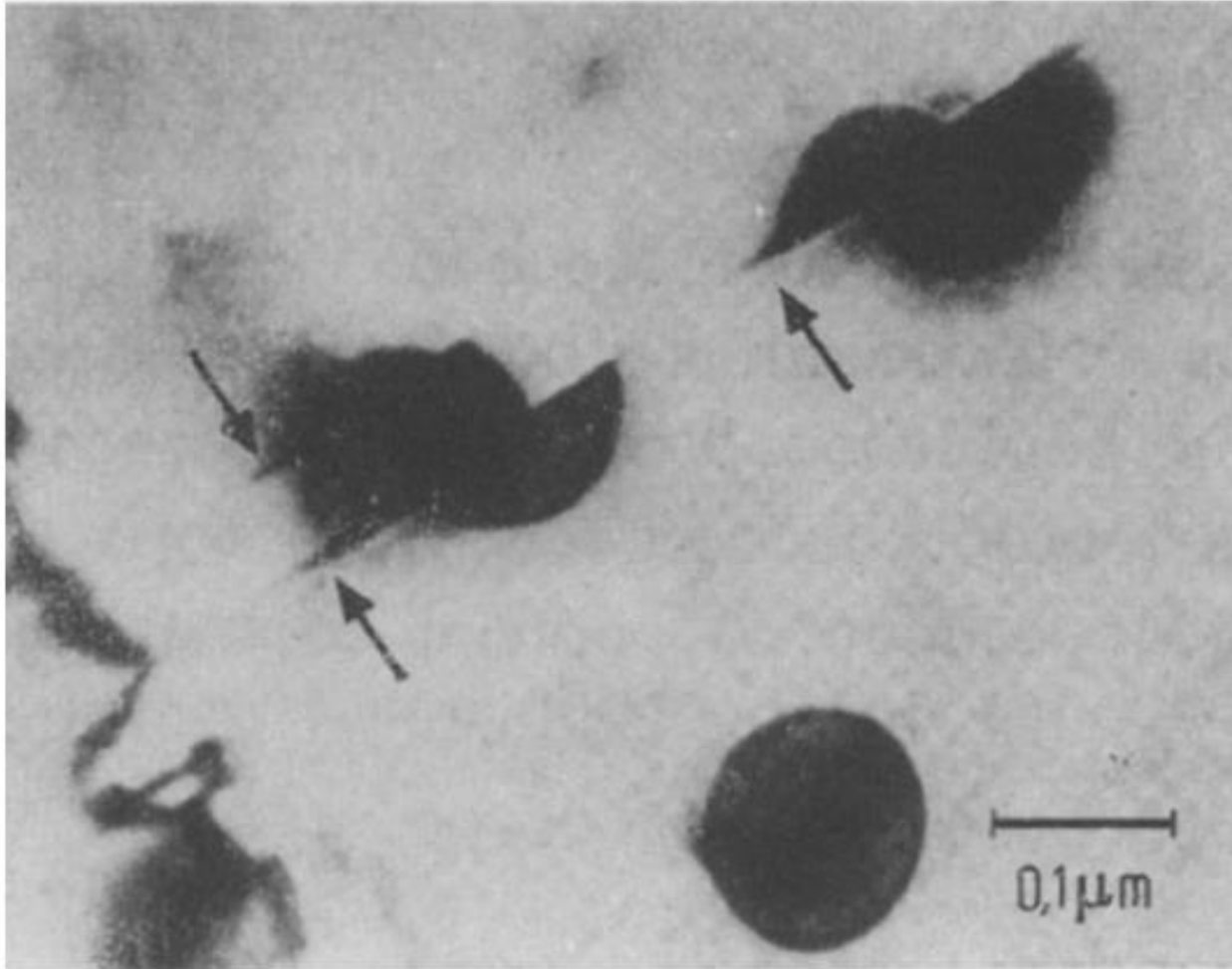
$$l_S^2 = \frac{\Delta\tau_c \cdot l_F^3}{Gb}$$

$$l_S = r \cdot \left( \frac{2\pi}{3f} \right)^{1/2}$$

$$\Delta\tau_{OR} = \sqrt{\frac{3}{16\pi}} \cdot Gb \cdot \frac{\sqrt{f}}{r} \cdot \sin^{3/2} \left( \frac{\theta_c}{2} \right)$$

$$\Delta\tau_{OR} \approx \frac{\sqrt{f}}{r}$$

# Interface strengthening: precipitate

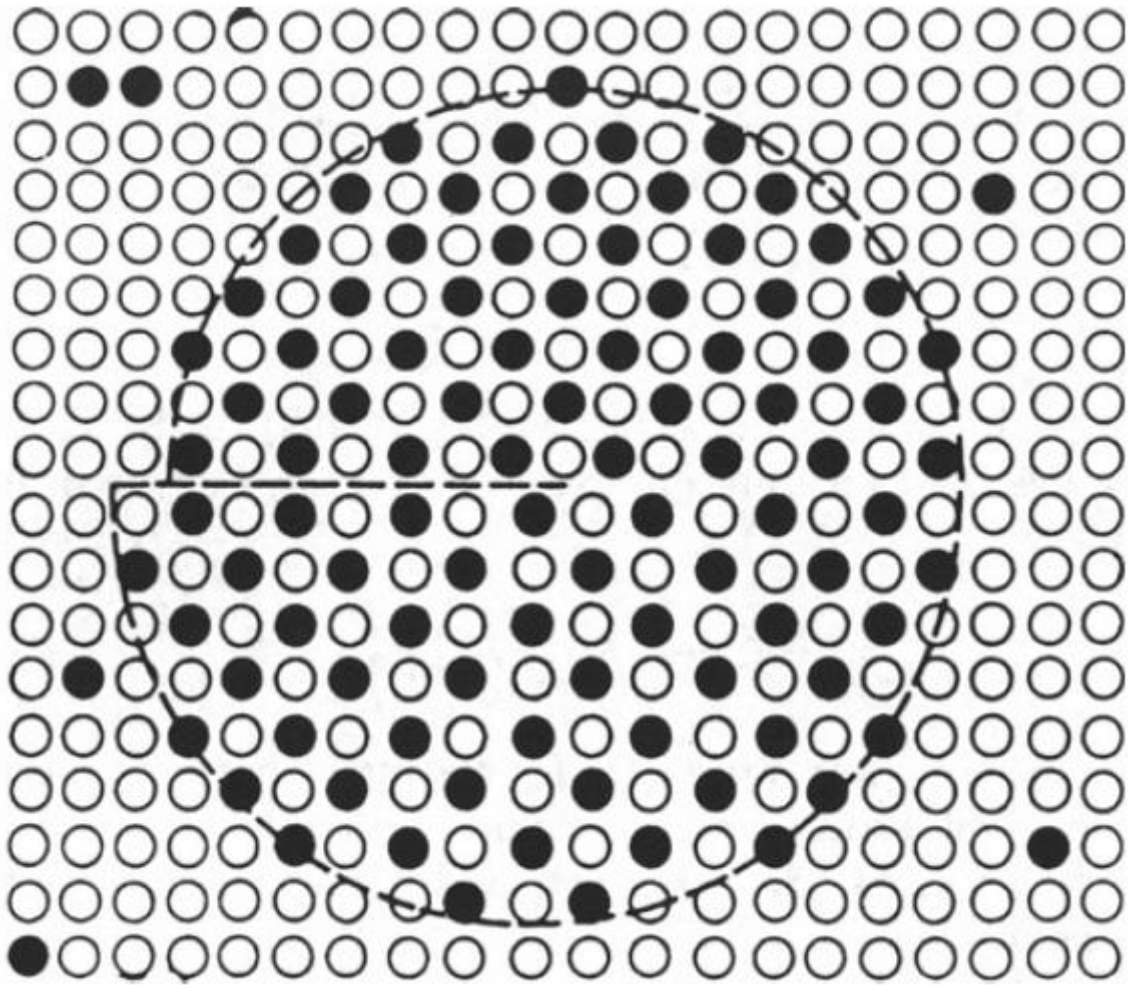


$$F_{pre} = \gamma_{pre} \cdot r$$

(N/m)

Fig. If a dislocation cuts a particle, the particle shears off. (a) Schematic; (b) observed in Ni19%Cr6%Al (aged 540h at 750°C and deformed 2% [6.25]).

# Interface strengthening: antiphase boundary

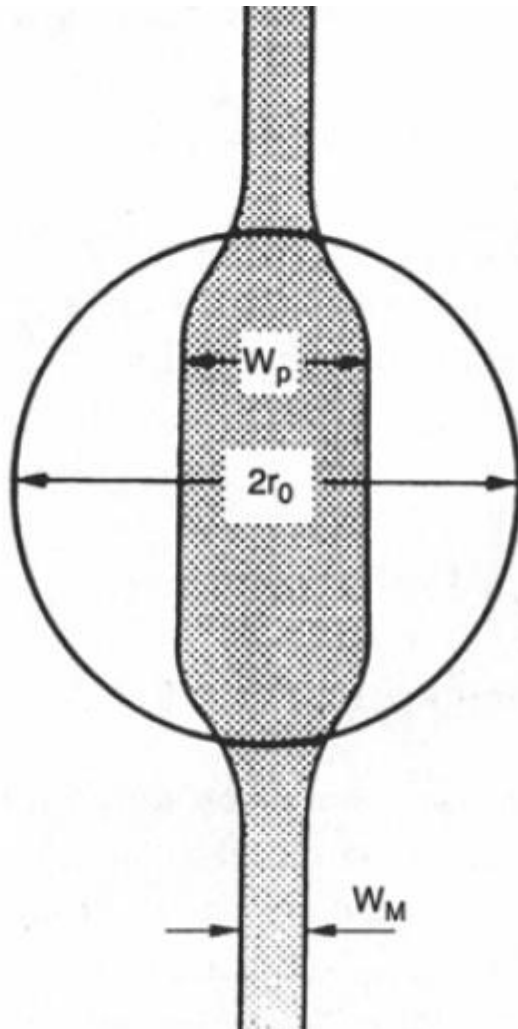


$$F_{anti} = \gamma_{anti} \cdot r$$

(N/m)

Fig. Intersecting an ordered particle produces an antiphase boundary.

# Interface strengthening: stacking fault



$$F_{st} = \gamma_{st} \cdot r$$

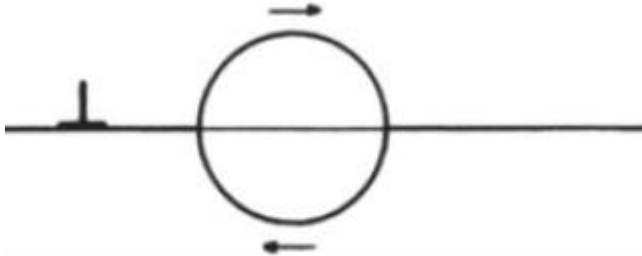
(N/m)

Fig. Change in dissociation width in a particle of different stacking fault energy.

# Critical shear stress of cutting

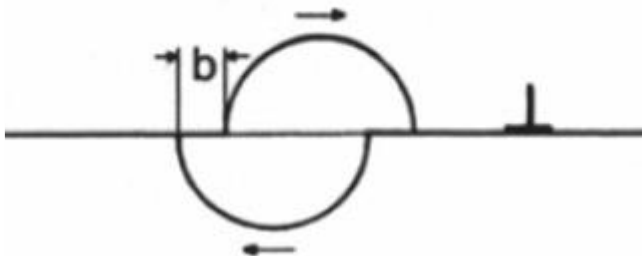
$$F_{\max} = F_{\max}^{pre} + F_{\max}^{anti} + F_{\max}^{st}$$

The presence of grain boundaries has an additional effect on the deformation behaviour of a material by serving as an effective barrier to the movement of glide dislocations.



$$F_{\max} = \Delta\tau_c \cdot b l_F$$

$$l_F = \left( \frac{Gb}{\Delta\tau_c \cdot c_F} \right)^{1/3}$$

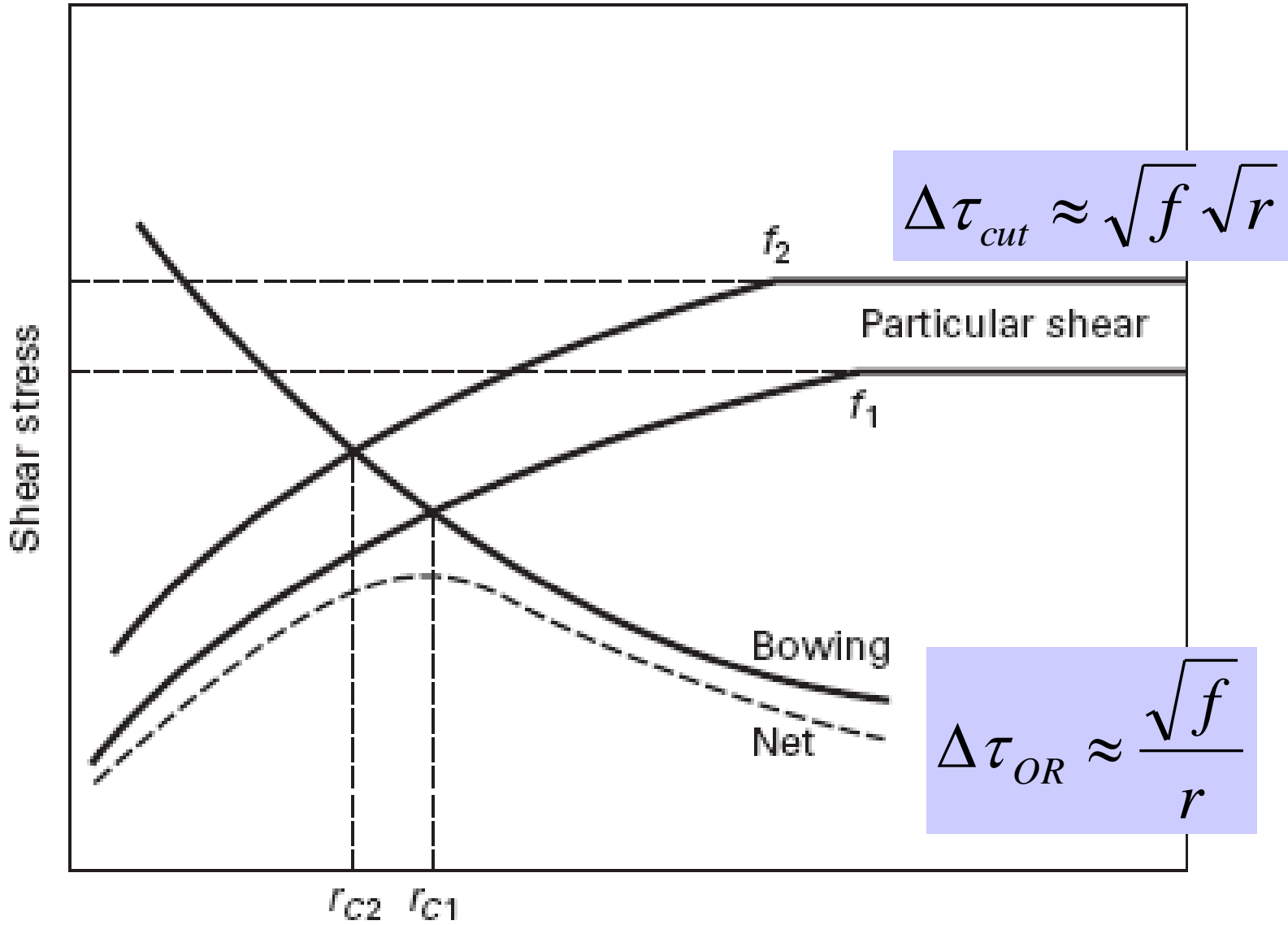


$$c_F = \frac{1}{l_F^2} = \frac{3f}{2\pi r^2}$$

$$\Delta\tau_{cut} \approx \sqrt{f} \sqrt{r}$$

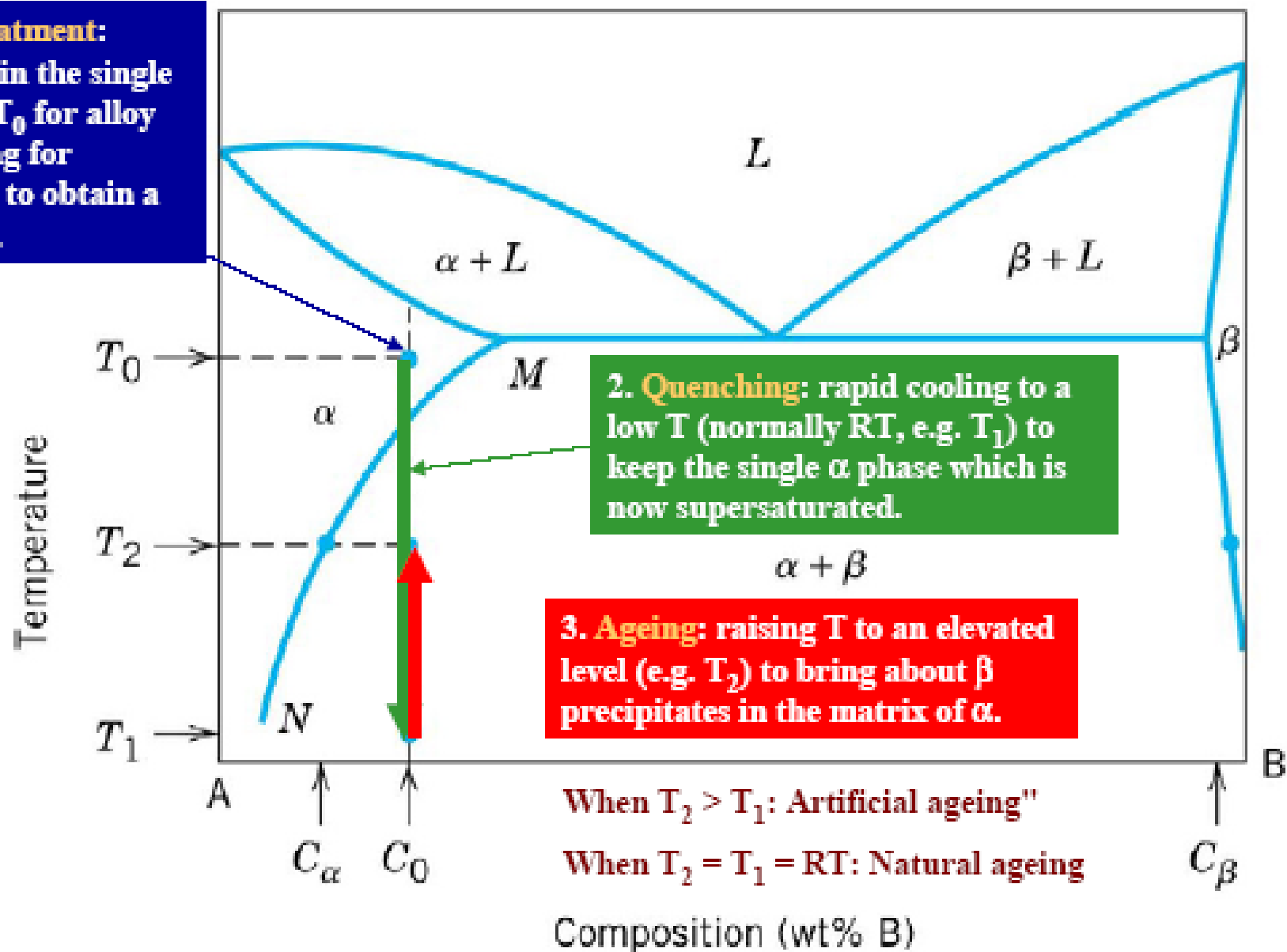
$$\Delta\tau_{cut} = \sqrt{\frac{3}{2\pi}} \cdot \sqrt{\frac{\gamma^3}{G}} \cdot \frac{1}{b^2} \cdot \sqrt{f \cdot r}$$

# By-passing and cutting mechanism



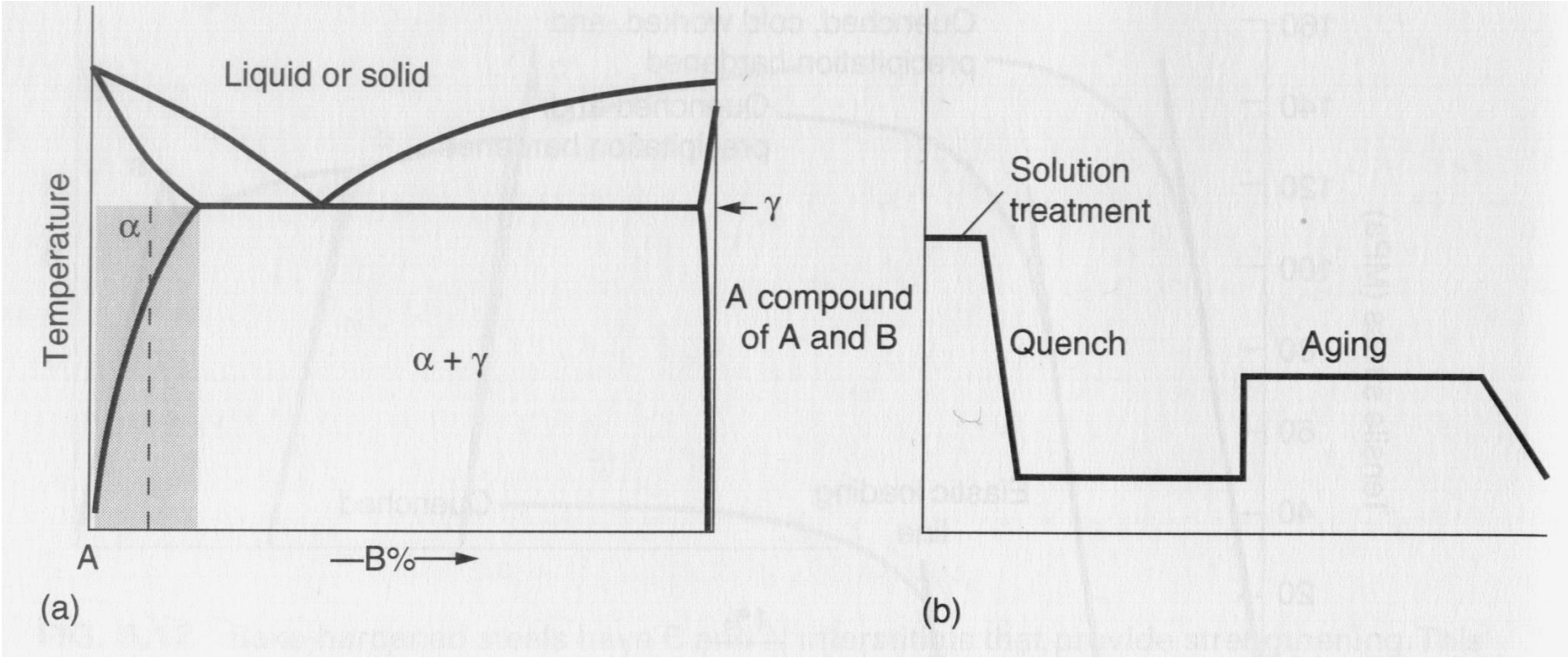
# Precipitation treatment

**1. Solution treatment:** heating to a  $T$  in the single  $\alpha$  region (e.g.  $T_0$  for alloy  $C_0$ ) and holding for sufficient time to obtain a single  $\alpha$  phase.



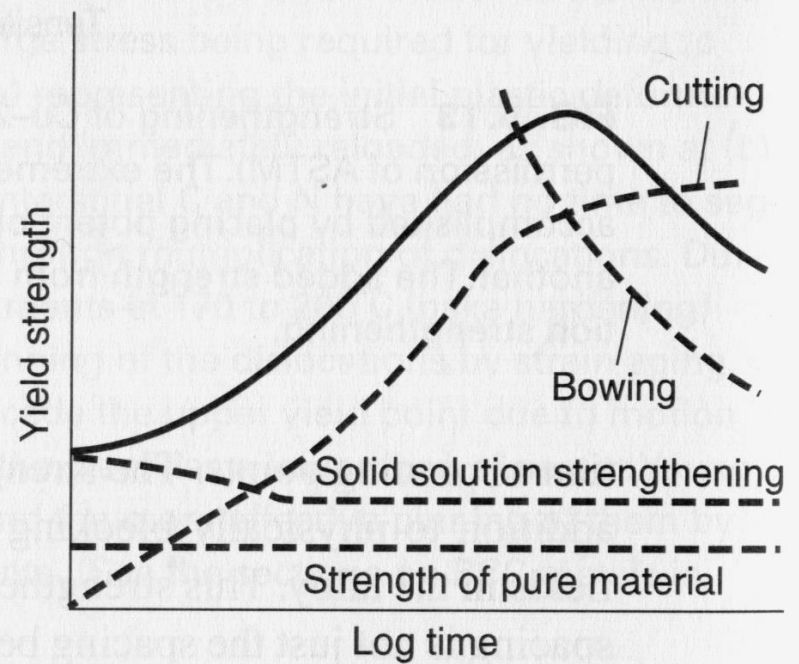
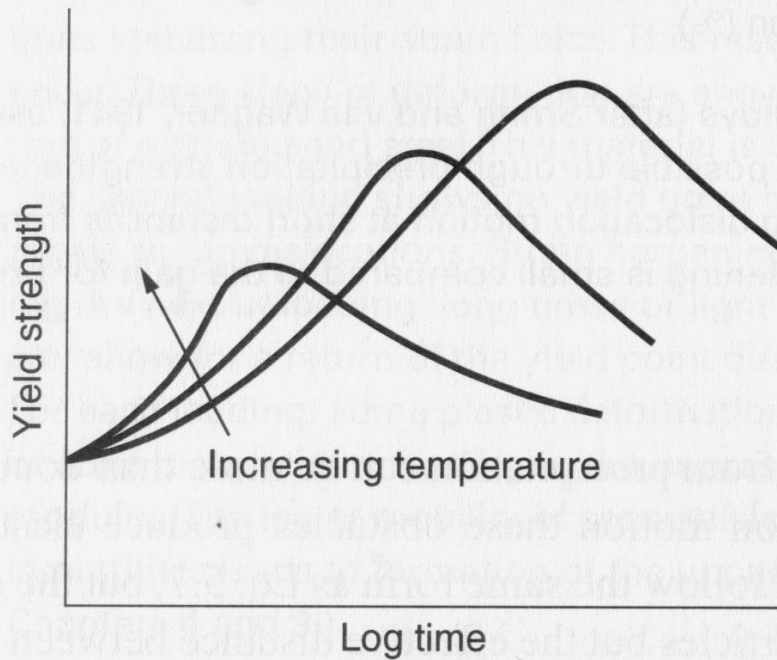


# Precipitation treatment



From K. Bowman, "mechanical behavior of materials", John Wiley & Sons Inc, 2004.

# Precipitation treatment

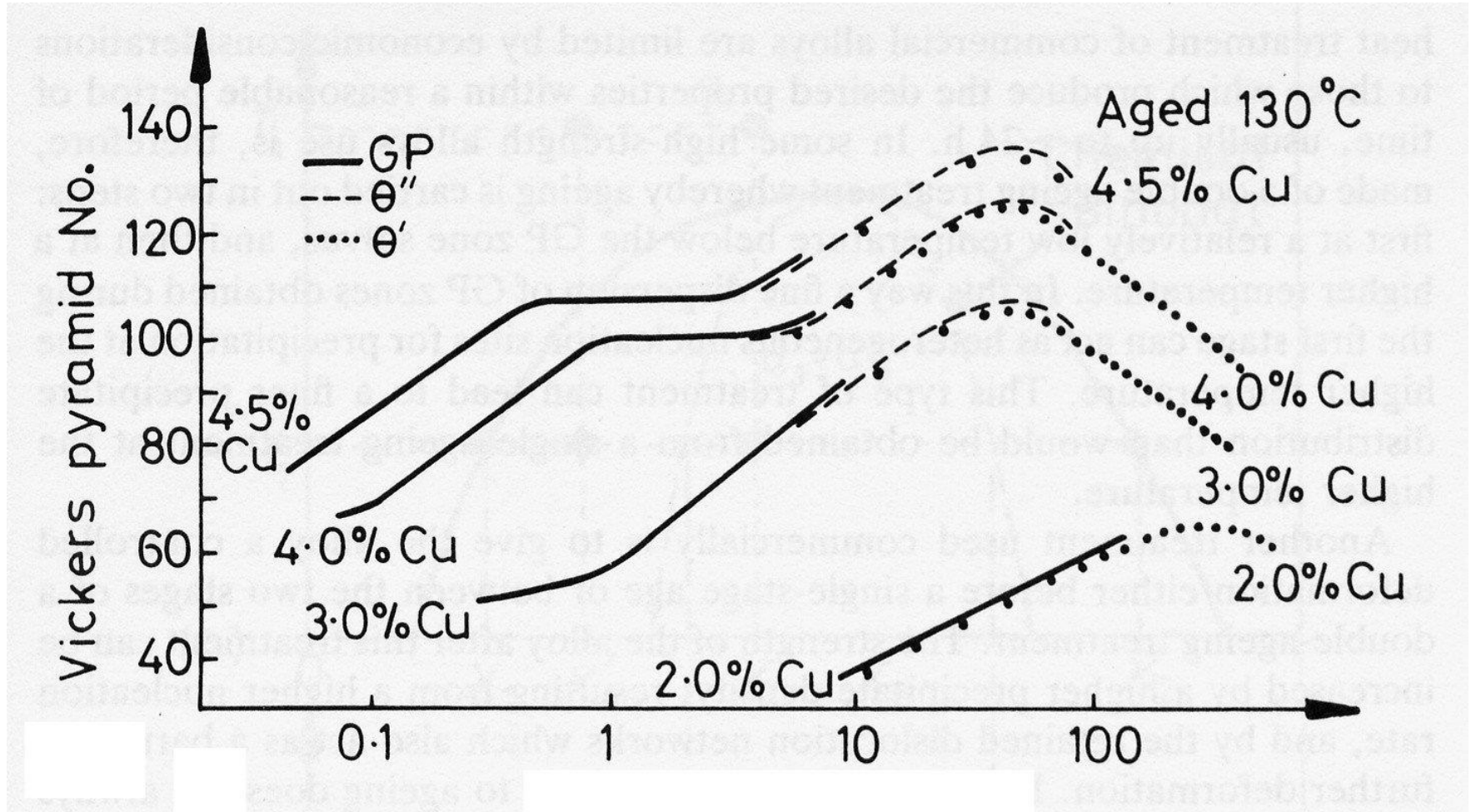


(c)

(d)

**FIG. 5.14** (a) Typical phase diagram for the potential of precipitation strengthening with  $\gamma$  being an intermetallic of elements A and B. (b). A possible process map for the alloy designated with a dashed line in part (a). (c) The effect of aging temperature on the time to peak strength in precipitation strengthened alloys. (d) The contributions from different strengthening mechanisms given as dashed lines add up to the total strength as a function of aging time.

# Al-Cu Precipitation



# Summary of Strengthening mechanisms

- Zero dimensional defect: Solid Solution strengthening  
solute atoms –dislocations interaction

$$\Delta\tau_{sol} \approx \varepsilon \cdot \Delta C$$

- 1 dimensional defect: Cold working strengthening  
dislocations –dislocations interaction

$$\Delta\tau_{dis} = \alpha Gb\sqrt{\rho}$$

- 2 dimensional defect: Grain boundary strengthening  
grain boundary –dislocations interaction

$$\Delta\tau_{gb} = kd^{-1/2}$$

- 3 dimensional defect: Second phase strengthening  
Precipitation/ Dispersion –dislocations interaction

$$\Delta\tau_c \approx \sqrt{f} \sqrt{r}$$

$$\Delta\tau_{OR} \approx \frac{\sqrt{f}}{r}$$