Mechanical Behaviour of Materials

Chapter 07-2

Fracture: Mechanisms

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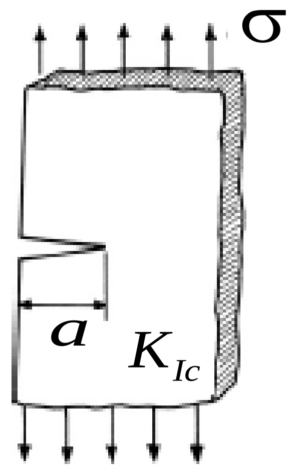
Facture Mechanics

A material fracture depends on temperature, the stress state and with time, and the environment.

Applied stress



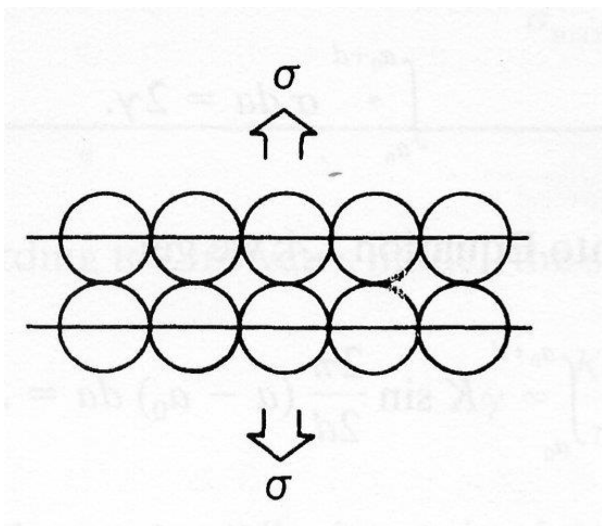
Flaw size

Fracture toughness

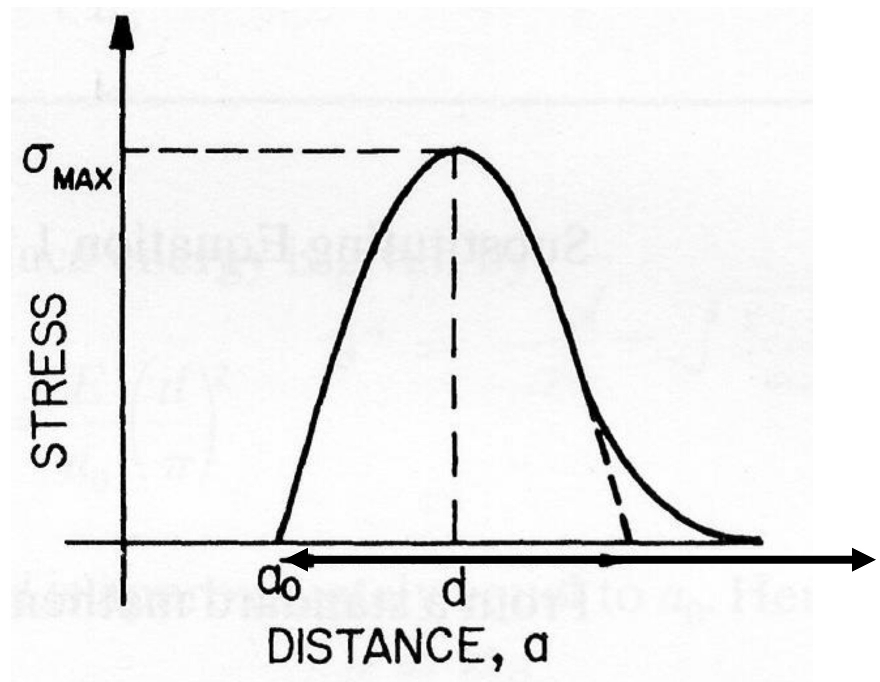
Theoretical cohesive strength

Under normal stress a material is to cleave, when the fracture surface is perpendicular to the applied stress.

The atoms are separated along the direction of the applied stress.



Stress required to separate two atomic layers



Theoretical cohesive strength

$$1.\ \sigma = \sigma_{th} \sin\left(\frac{2\pi}{d} x\right)$$

$$\sigma_{th} \approx \frac{E}{2\pi} \approx \frac{E}{10}$$

$$2.\ \frac{dx}{a_0} = d\varepsilon$$

$$3.\ \frac{d\sigma}{d\varepsilon} = \frac{d\sigma}{dx / a_0} = E$$

$$\sigma_{th} = \frac{d}{2\pi a_0} E$$

$$a_0 \frac{d\sigma}{dx} = \frac{2\pi}{d} \sigma_{th} \cos\left(\frac{2\pi x}{d}\right)$$

$$\frac{d\sigma}{dx} = \left(\frac{2\pi}{d} \sigma_{th}\right)_{x \to 0}$$

Theoretical facture strength: energy approach

$$U_{fracture} = \int_0^{d/2} \sigma_{th} \sin\left(\frac{2\pi x}{d}\right) dx = \sigma_{th} \frac{d}{\pi}$$

$$\downarrow \quad d = \frac{2\pi a_0}{E} \sigma_{th}$$

$$\sigma_{th}^2 = \frac{\gamma E}{a_0}$$

$$\sigma_{th} = \sqrt{\frac{E\gamma}{a_0}}$$

σ_{th}: maximum stress at the end of the major axis

σ_a: applied stress normal to the major axis

a: half major axis

b: half minor axis

Theoretical Cleavage Strength

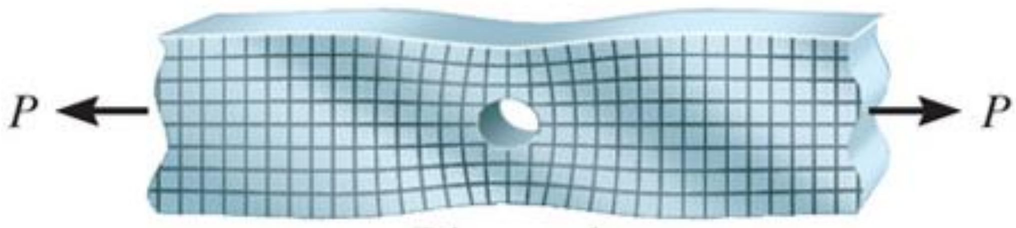
Table 7.1 Theorectical Cleavage Stresses According to Orowan's Theory*

Element	Direction	Young's Modulus (GPa)	Surface Energy (J/m2	σ_{max} (GPa)	σ_{max}/E
σ-Iron	<100>	132	2	30	0.23
	<111>	260	2	46	0.18
Silver	<111>	121	1.13	24	0.20
Gold	<111>	110	1.35	27	0.25
Copper	<111>	192	1.65	39	0.20
	<100>	67	1.65	25	0.38
Tungsten	<100>	390	3.00	86	0.22
Diamond	<111>	1,210	5.4	205	0.17

* Adapted with permission from A. Kelly, *Strong Solids*, 2nd ed. (Oxford, U.K.: Clarendon Press, 1973), p. 73.

Crack-initiated fracture: stress concentration

The fundamental requisite for the propagation of a crack is that the stress at the tip of the crack must exceed the theoretical cohesive strength of the material.



$$k_t = \frac{\sigma_{max}}{\sigma_a}$$

Actual stress distribution

The stress concentration factor (SCF) is as the ratio of the maximum stress to the applied stress.

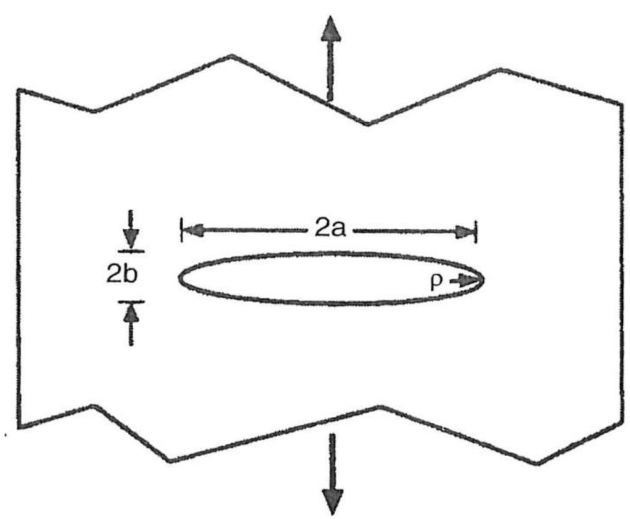
Stress concentration factor (stress approach)

Inglis: The stress rises dramatically near the hole and has a maximum value at the edge of the hole. The maximum value is given:

$$\frac{\sigma_{\max}}{\sigma_a} = 1 + 2\frac{a}{b}$$

External applied stress ← σ_a

$1 + 2\frac{a}{b}$ → stress concentration factor



$$\sigma_{\max} = \sigma_a \left(1 + 2\sqrt{\frac{a}{\rho}} \right)$$

$$\rho = \frac{b^2}{a}$$

the radius of curvature at the end of the ellipse

$$\frac{\sigma_{\max}}{\sigma_a} \approx 2\sqrt{\frac{a}{\rho}} \qquad a >> \rho$$

$$k_t = 2\sqrt{\frac{a}{\rho}}$$

stress concentration factor

$$k_t = \frac{\sigma_{\max}}{\sigma_a}$$

Stress concentration factor (stress approach)

$$\sigma_{max} = \sigma_a \left(1 + 2 \frac{a}{b} \right)$$

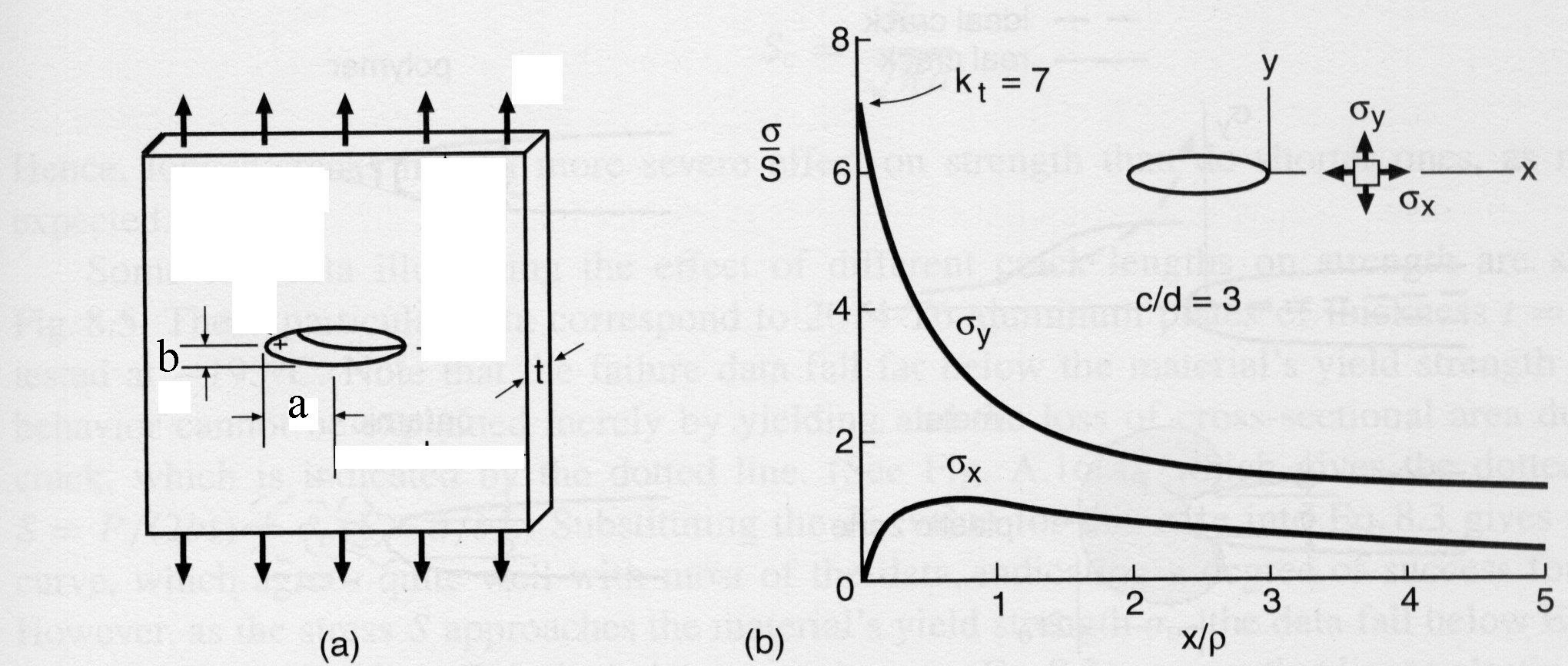
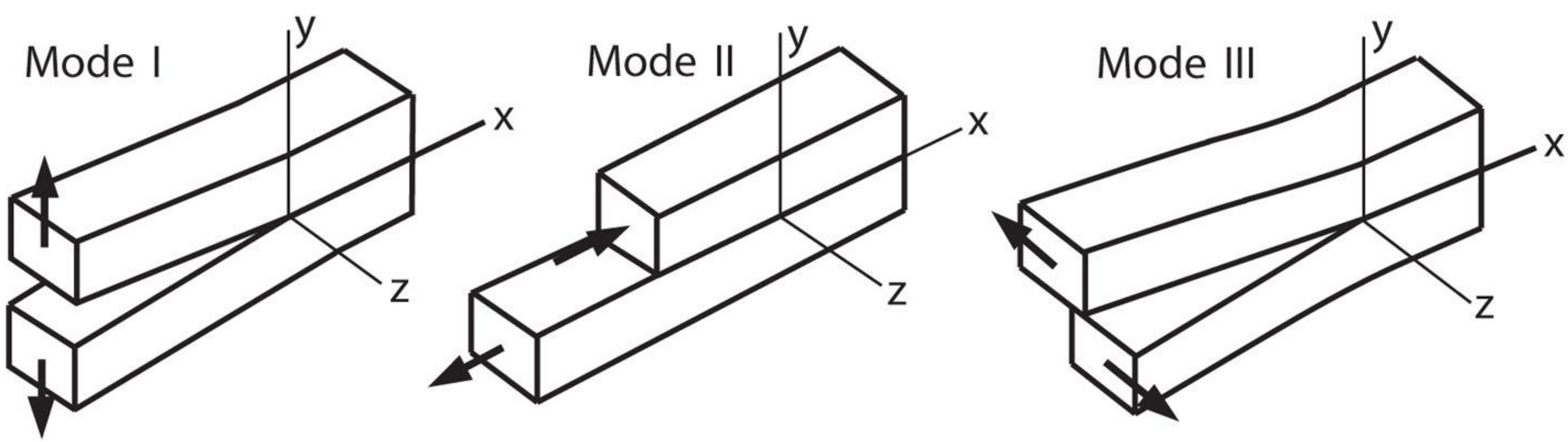


Figure 8.3 Elliptical hole in a wide plate under remote uniform tension, and the stress distribution along the *x*-axis near the hole for one particular case.

Facture modes



I. Opening or
tensile mode

II. Sliding or
In-plane shear mode

III. Tearing or
antiplane shear mode

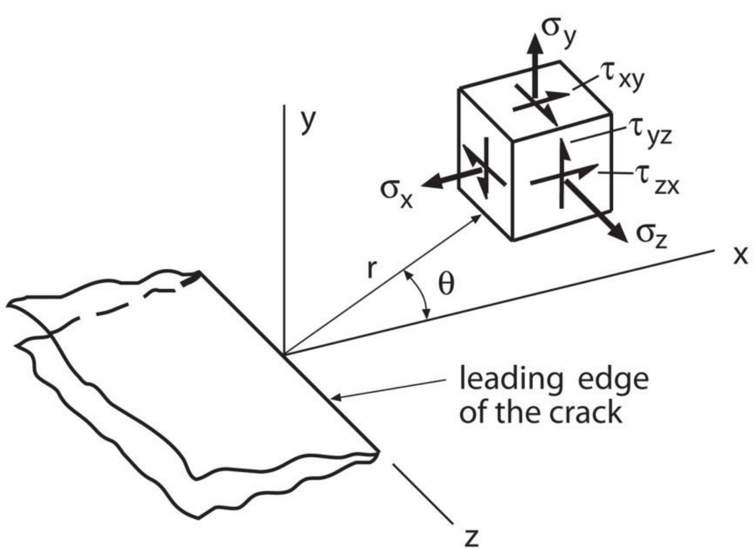
Irwin's fracture analysis: stress approach

Irwin proposed the stress state around an Infinitely sharp crack in a semi-infinite elastic solid

$$\sigma_x = \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\sigma_y = \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$



$$\sigma_z = 0$$ plane stress condition

$$\sigma_z = \upsilon\left(\sigma_x + \sigma_y\right)$$ plane strain condition

$$\tau_{yz} = \tau_{zx} = 0$$

Stress intensity factor

Stress intensity factor in a semi-infinite body is given:

$$K = \sigma\sqrt{\pi a}$$

Stress intensity factor for finite body is given:

$$K = \sigma\sqrt{\pi a} \cdot f$$

f depends on the specimen geometry
and is >1 for small crack

Fracture occurs when K reaches a critical value, K_c, fracture toughness.

$$\sigma_f = \frac{K_c}{f(\pi a)^{1/2}} \qquad f = 1 \qquad K_c = \sqrt{EG_c}$$

Comparison between k_t and K

K (the stress intensity factor): provides a complete description of the state of stress, strain and displacement over some region of the body, is dependent of the crack length and the geometry of the body.

$$K = \sigma \cdot \sqrt{\pi a} \cdot f\left(\frac{a}{w}\right)$$

k_t (the stress concentration factor): determines the magnitude of the maximum stress at a single point.

$$k_t = 2\sqrt{\frac{a}{\rho}}$$

Griffith crack theory (energy criterion)

thickness

$$U_{\text{surf}} = 2\,a \cdot t \cdot 2\gamma$$

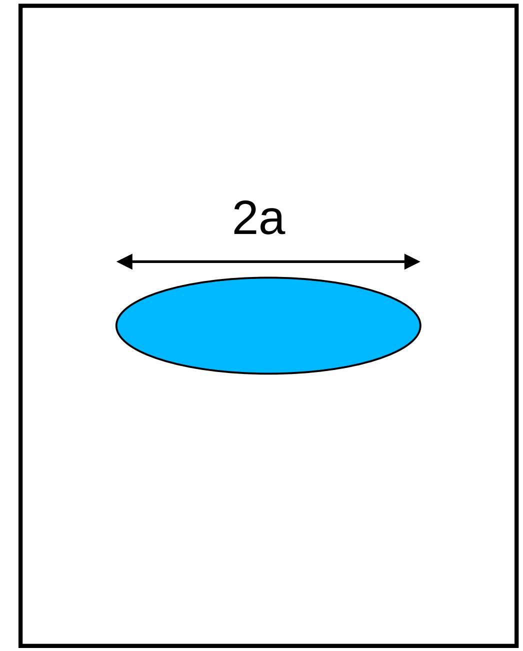
$$U_{\text{el}} = \left(\frac{\sigma^2}{2E}\right)\left(2\pi a^2 t\right) = \frac{\pi\, a^2 t \sigma^2}{E}$$

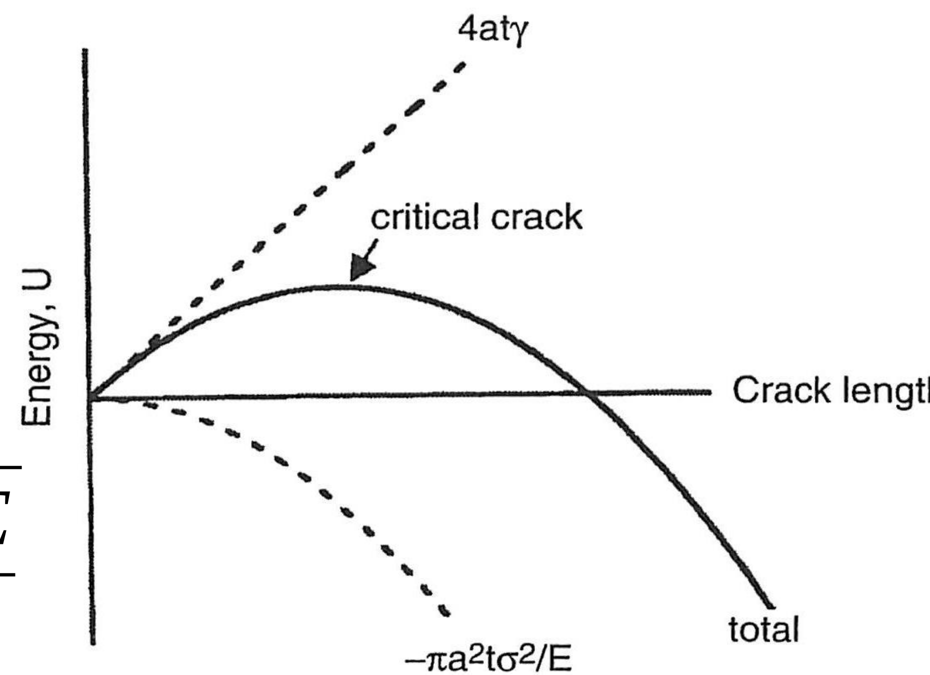
$$\Delta U = 4at\gamma - \frac{\pi\, a^2 t\sigma^2}{E}$$

Equilibrium condition

$$\frac{d\Delta U}{da} = 4t\gamma - \frac{2\pi\, a^2 t\sigma^2}{E} = 0$$

Fracture stress $\sigma_f = \sqrt{\frac{2\gamma E}{\pi a}} = \sqrt{\frac{GE}{\pi a}}$



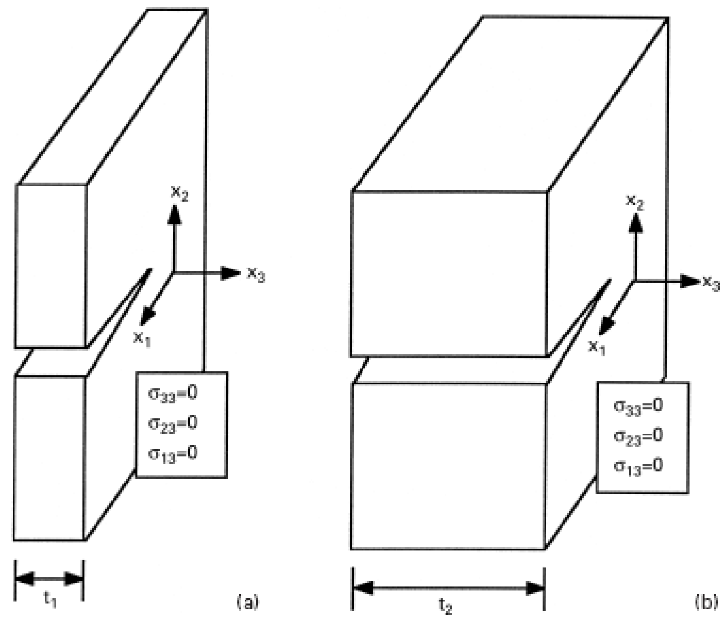


Griffith theory-conti.

Plane strain

Plane stress

$$\sigma_f = \sqrt{\frac{GE}{\pi a}}$$



$$\sigma_f = \sqrt{\frac{GE}{(1-\nu^2)\pi a}}$$

U_{el}: elastic energy of body with crack

$U_{surface}$: surface energy of body with crack

σ: applied stress

a: one-half crack length

t: thickness

E: modulus of elasticity

γ: specific surface energy

Orowan theory: energy including plastic energy

$$\sigma_f = \frac{K_c}{\sqrt{\pi a}}$$

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}} \longrightarrow \sigma = \sqrt{\frac{EG_c}{\pi a}}$$

$$G_c = 2\left(\gamma_s + \gamma_p\right)$$

Including the plastic work in generating the fracture surface

$$K_c = \sqrt{EG_c} = \sigma_c \sqrt{\pi\left(a + r_p\right)}$$

Fracture toughness

Plastic zone size

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

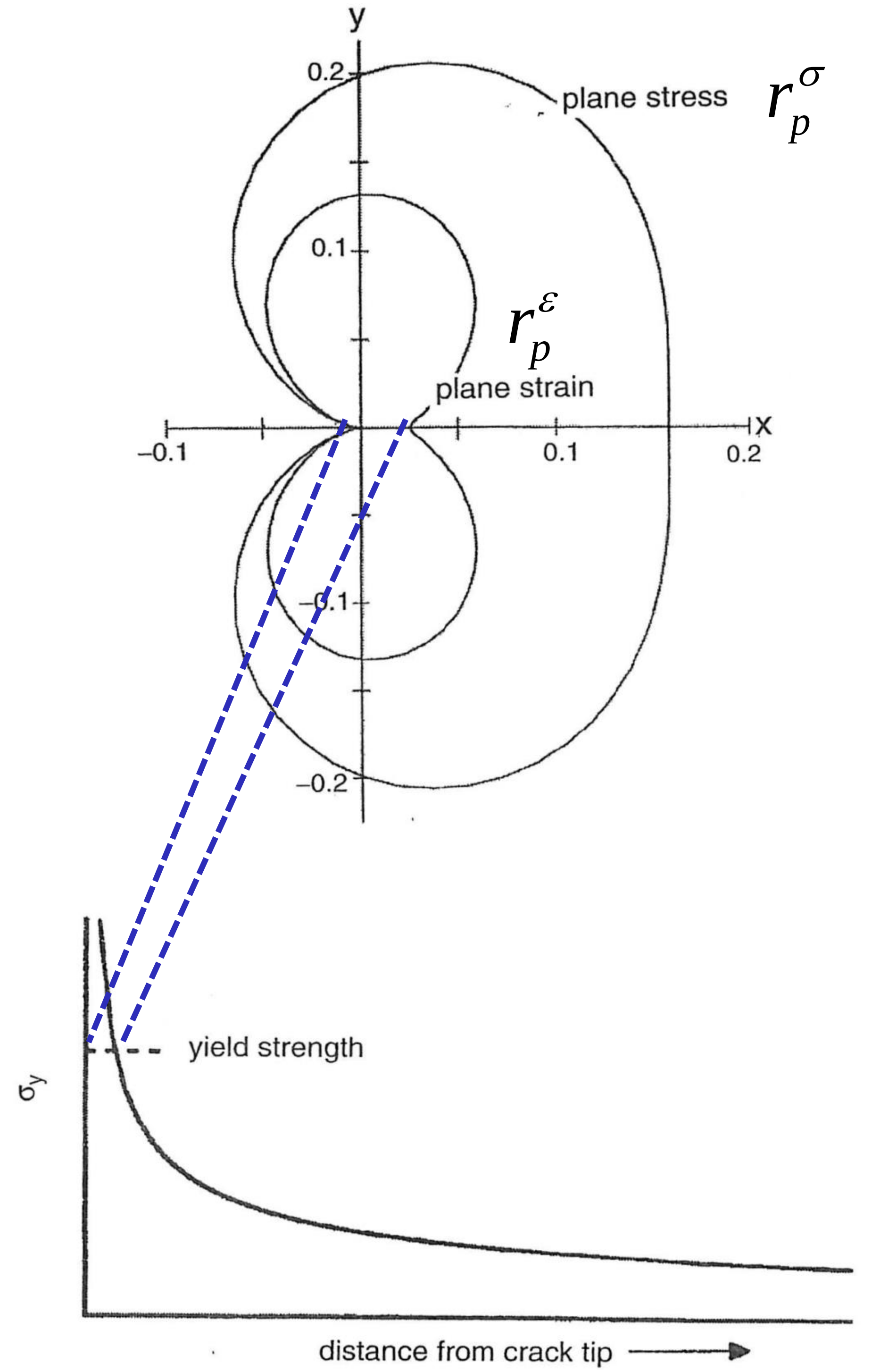
at $\theta = 0$ $\quad \sigma_y = \frac{K}{\sqrt{2\pi r}}$

$$r_p = \frac{\left(\frac{K_c}{Y}\right)^2}{2\pi}$$

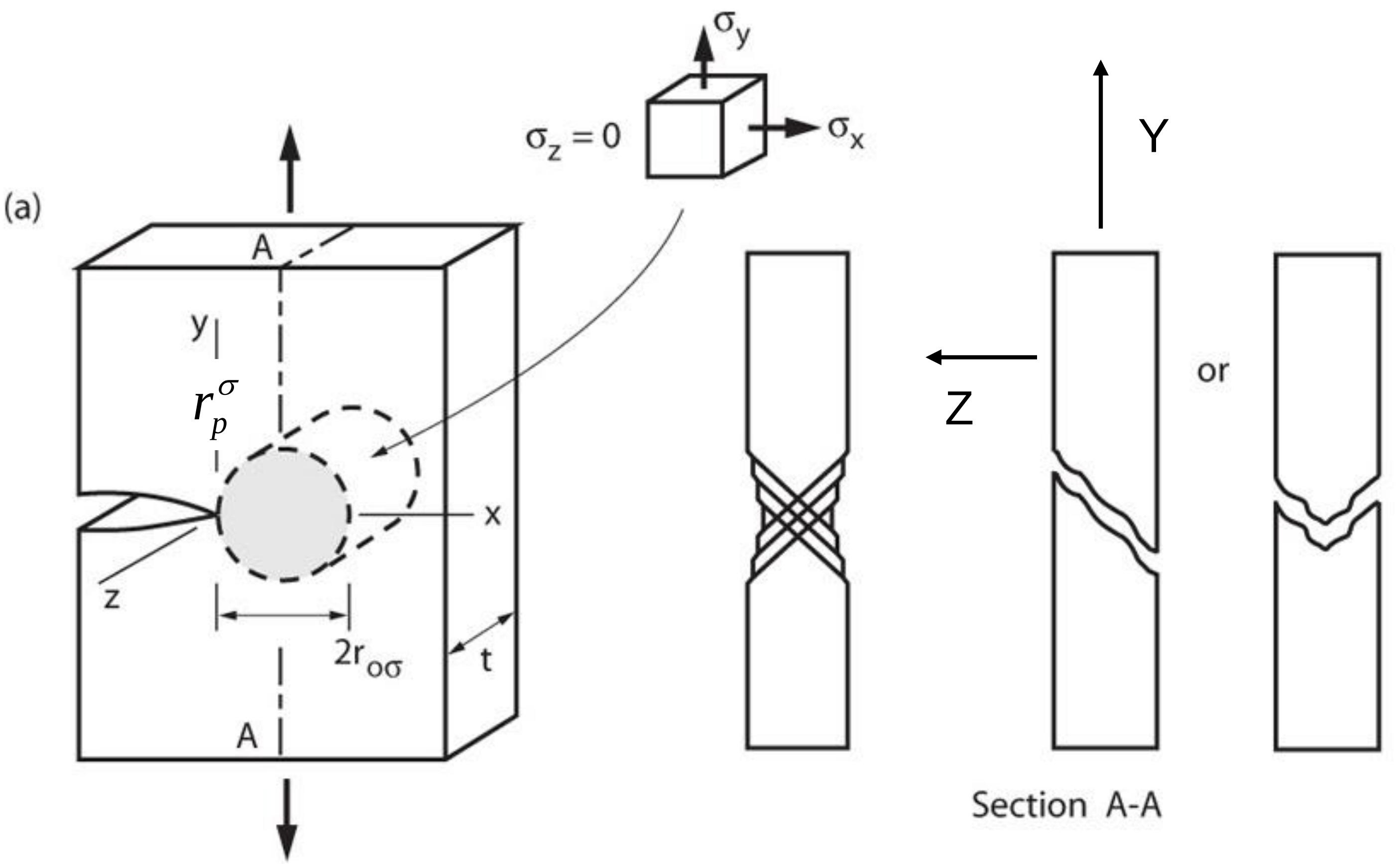
Plane stress

$$r_p = \frac{\left(\frac{K_c}{Y}\right)^2}{6\pi}$$

Plane strain



Slant fracture : Plane stress



$$\sigma_x \neq 0$$

$$\sigma_y \neq 0$$

$$\sigma_z = 0$$

$$\varepsilon_x \neq 0$$

$$\varepsilon_y \neq 0$$

$$\varepsilon_z \neq 0$$

$$\tau_{\max} = ?$$

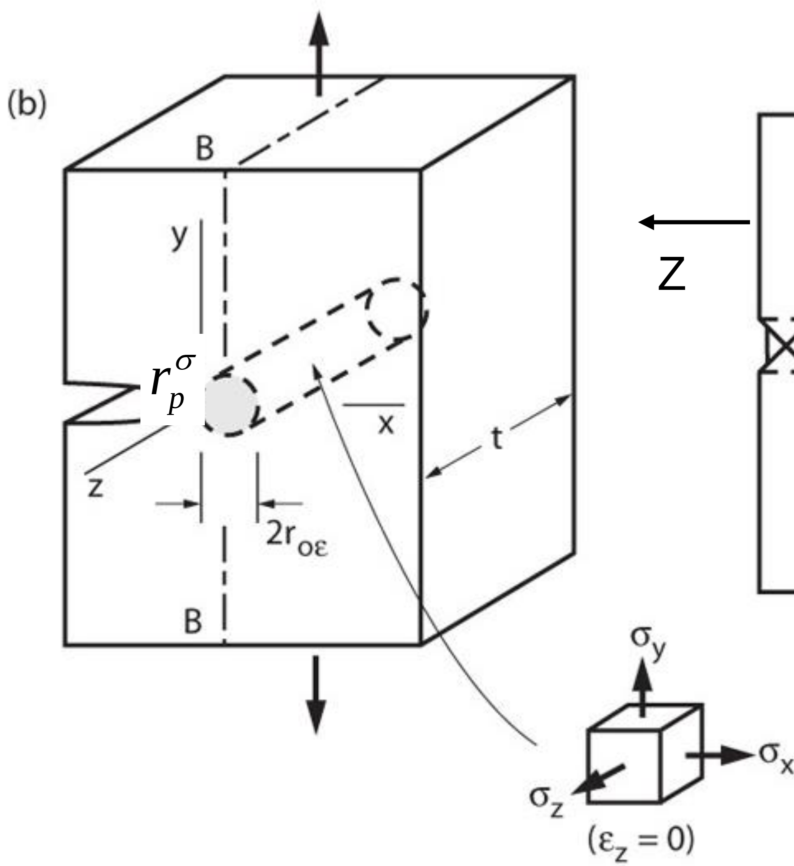
$$\sigma_y > \sigma_x$$

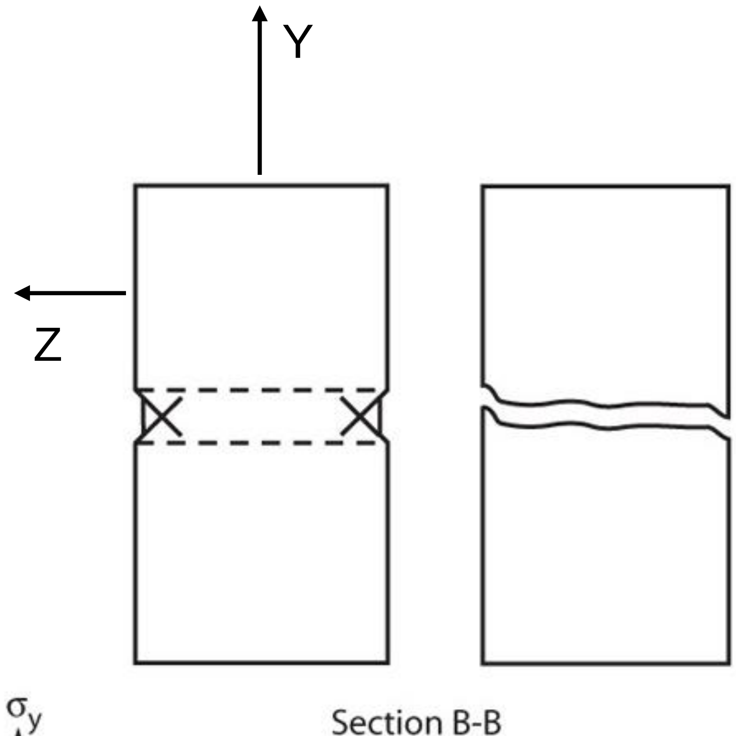
$$\sigma_z = 0$$

Remarks:

Biaxial stress state

Flat fracture: Plane strain





$$\sigma_x \neq 0$$

$$\sigma_y \neq 0$$

$$\sigma_z = \nu\left(\sigma_x + \sigma_y\right)$$

$$\varepsilon_x \neq 0$$

$$\varepsilon_y \neq 0$$

$$\varepsilon_z = 0$$

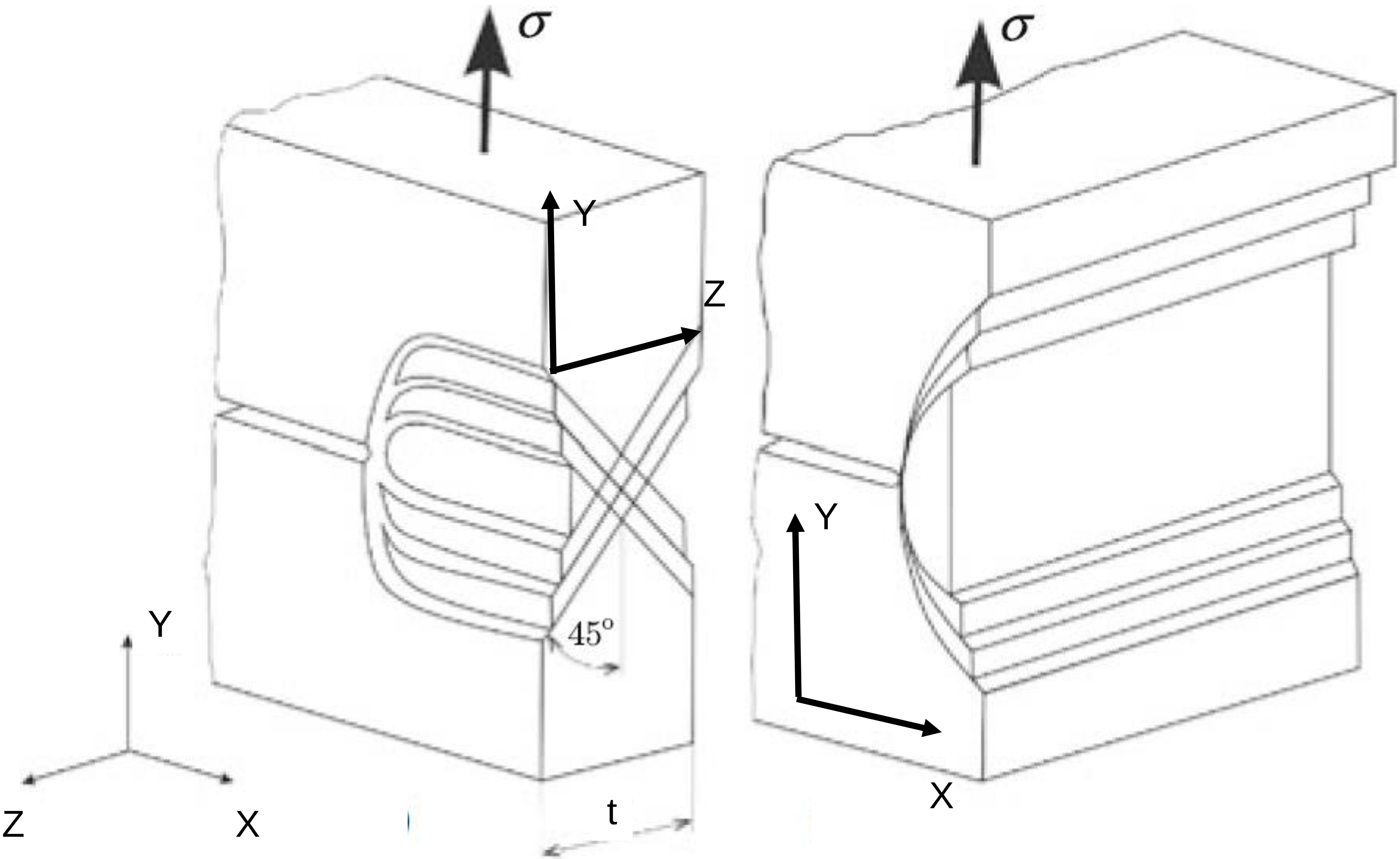
$$\tau_{\max} = ?$$

$$\sigma_y > \sigma_z > \sigma_x$$

Remarks:

1. The triaxial stress state of plane strain reduces the plastic zone size in comparison to the plane stress zone size.
2. The triaxial stress state is pronounced at the boundary between the plastic and elastic zones.

Fracture plane: Plane stress and Plane strain



Plane strain

Plane stress

Effect of thickness on K_c

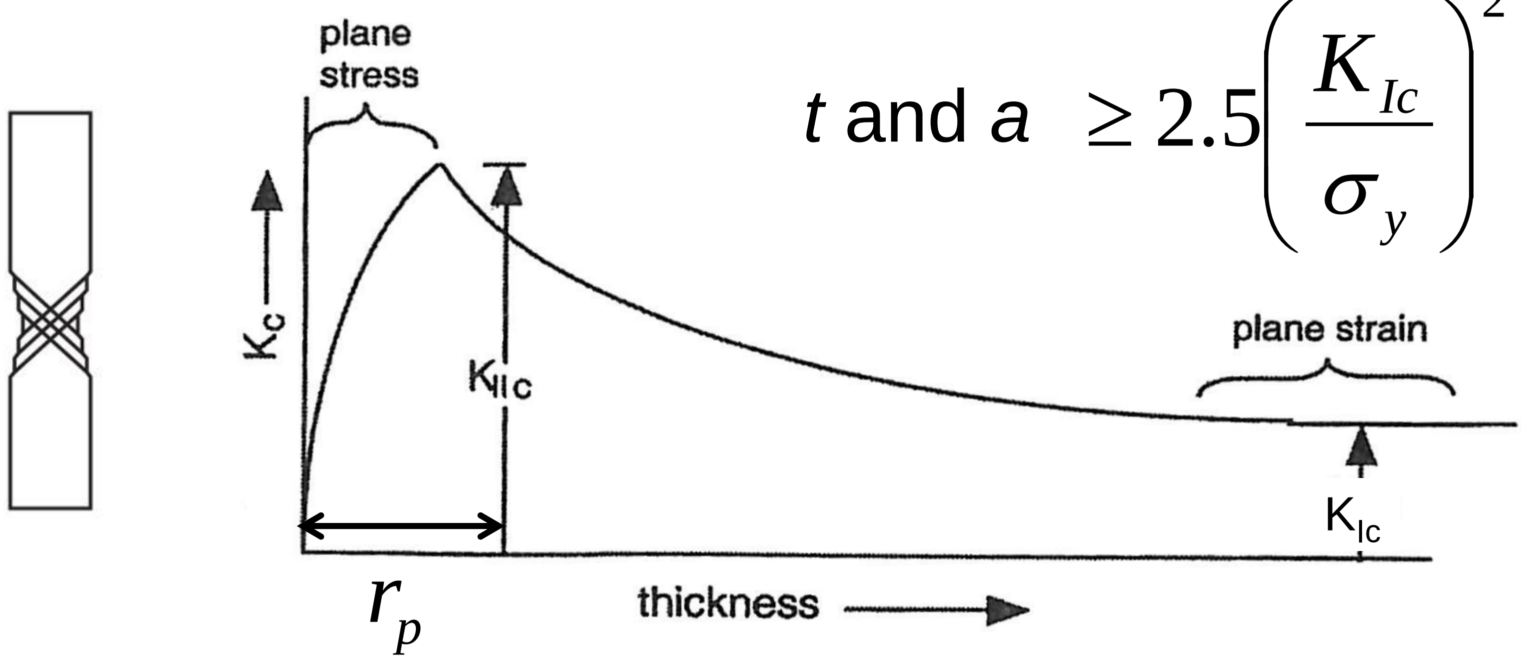
$$K_c = \sqrt{EG_c}$$

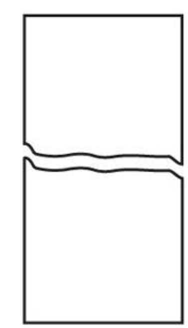
$$G_c = 2(\gamma_s + \gamma_p)$$

$$t \leq 2 \cdot r_p$$

The thickness of the specimen should be much greater than the radius of the plastic zone for plane stress:

$$t \text{ and } a \geq 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$





Fracture toughness testing

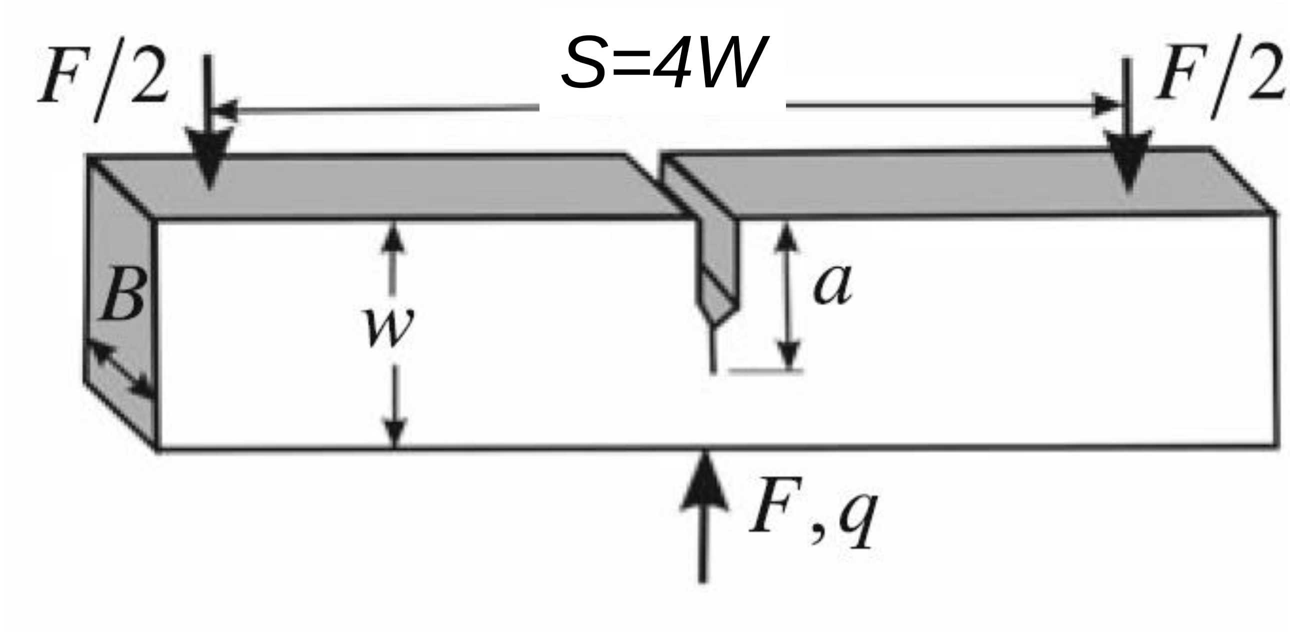
The following are the fracture toughness parameters commonly obtained from testing

- **K (stress intensity factor)** can be considered as a **stress-**based estimate of fracture toughness. K depends on geometry (the flaw depth, together with a geometric function, which is given in test standards for each test specimen geometry).

- **CTOD (crack-tip opening displacement)** can be considered as a **strain-**based estimate of fracture toughness. However, it can be separated into elastic and plastic components. The elastic part of CTOD is derived from the stress intensity factor, K. The plastic component is derived from the crack mouth opening displacement (measured using a clip gauge).

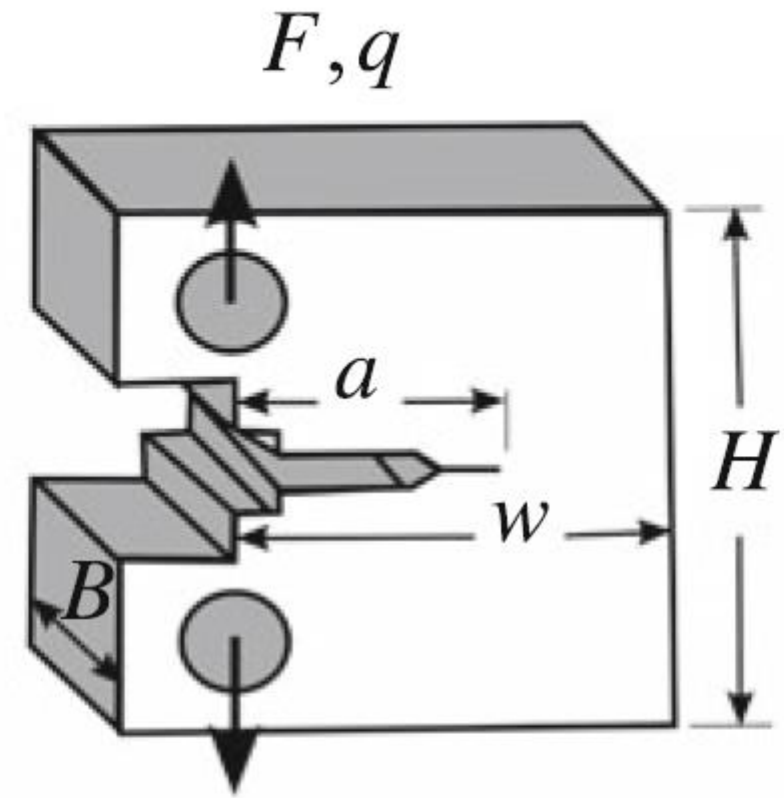
- **J (J-integral)** is an **energy-**based estimate of fracture toughness. It can be separated into elastic and plastic components. As with CTOD, the elastic component is based on K, while the plastic component is derived from the plastic area under the force-displacement curve.

Plane-strain fracture testing of metals:
single edge notch bend (SENB or three-point bend)

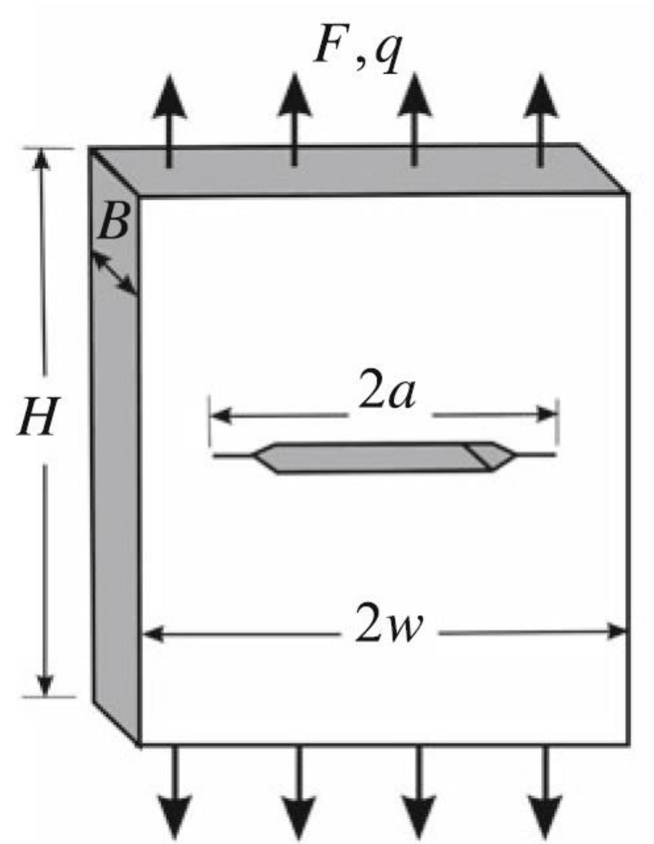


$$B \geq 2.5\left(\frac{K_{IC}}{\sigma_y}\right)^2$$

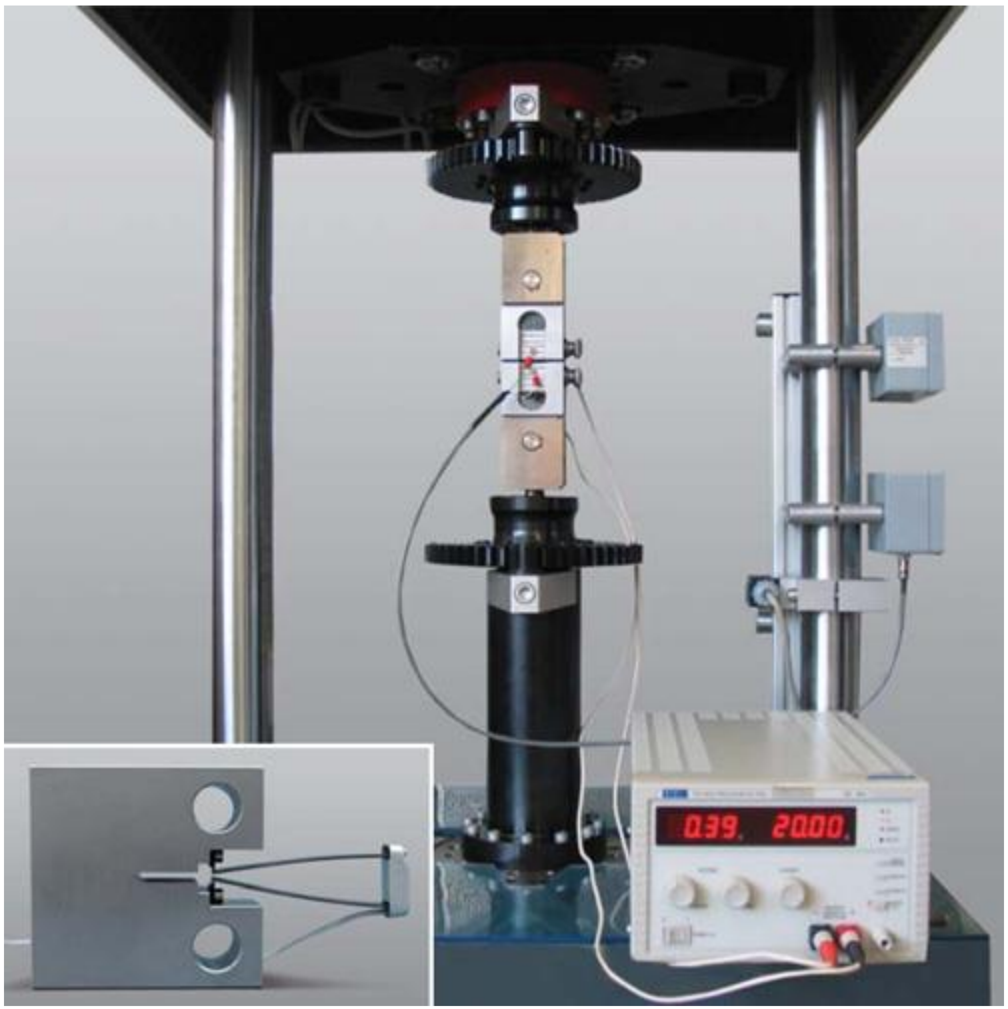
Plane-strain fracture testing of metals: compact tension (CT) specimen

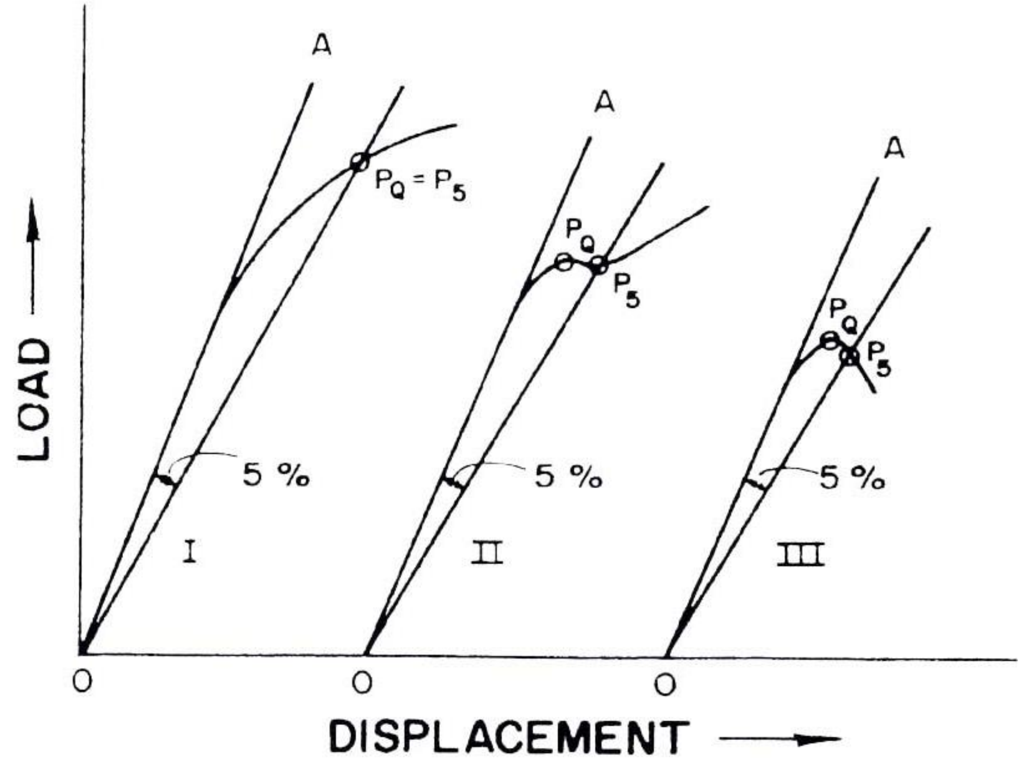


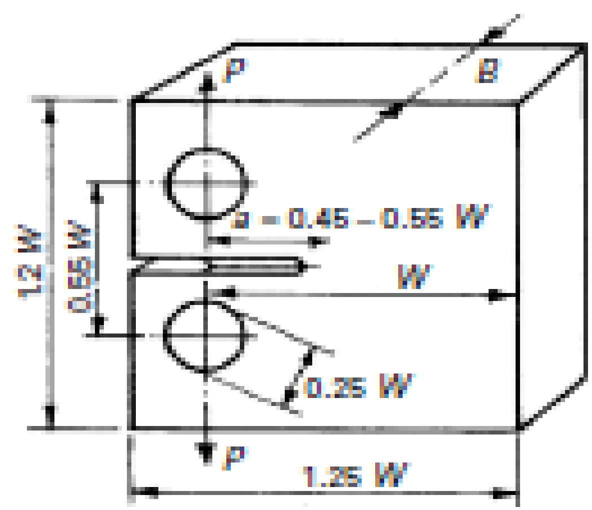
Plane-strain fracture testing of metals



Measurement of fracture toughness





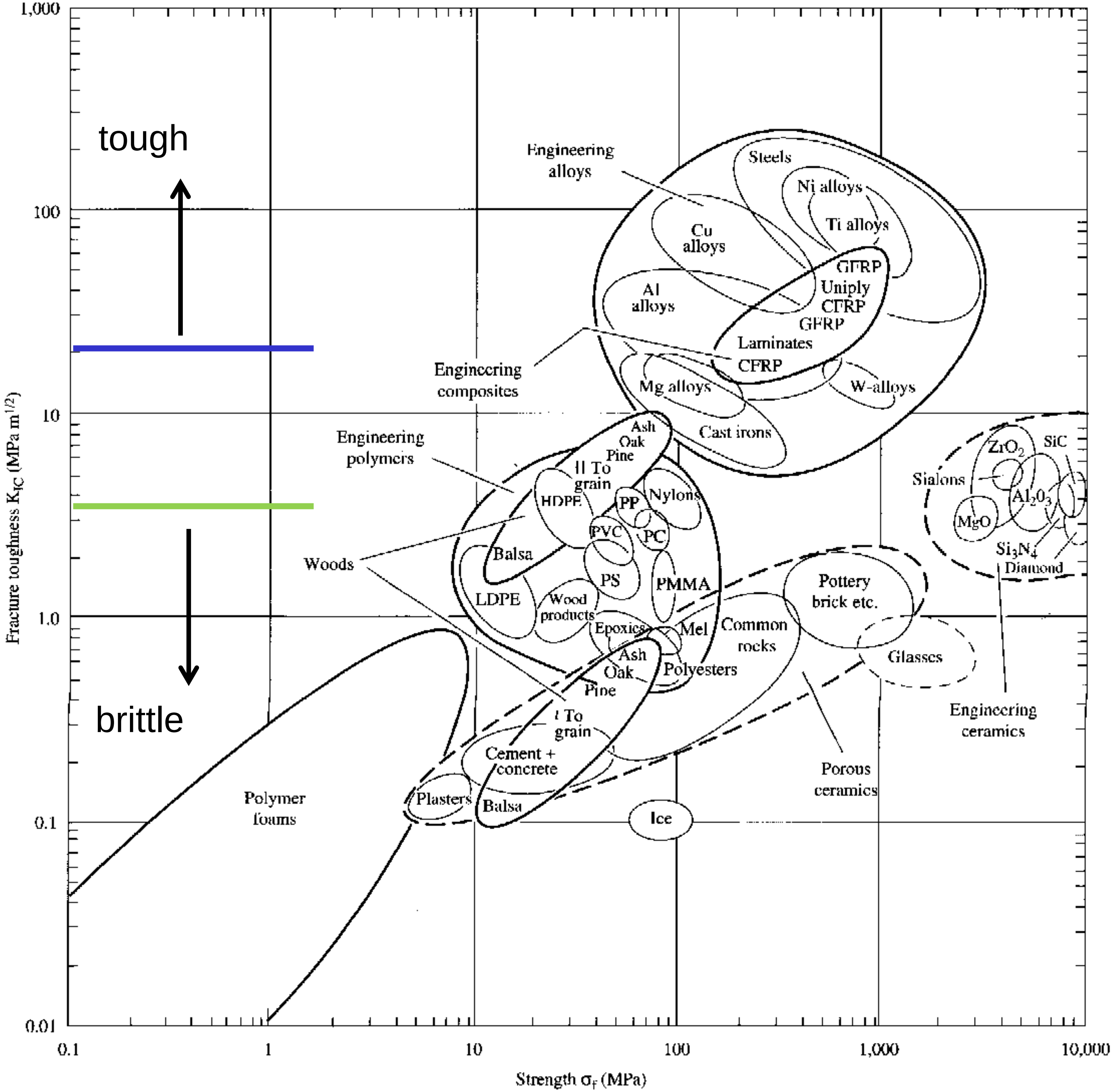


Fracture toughness

TABLE 7.1 Typical Ranges of Plane Strain Fracture Toughness and Yield Strength for Several Materials at Room Temperature

Material	K_{Ic} (MPa$\sqrt{m}$)	Y (MPa)
Al 2000 series	24–40	300–450
Al 7000 series	25–35	400–600
Ti-6A1-4V alloys	50–110	800–1100
4340 steel	55–105	1300–1700
Maraging steels	40–80	1400–2300
Alumina (Al_2O_3)	3–5	—
Boron carbide (BC)	4–6	—
Silicon nitride (Si_3N_4)	4–8	—
Silicon carbide (SiC)	2–5	—
Tetragonal zirconias (doped ZrO_2)	4–10	—
Epoxies	0.5–0.8	—
Borosilicate glass	0.5–1	
Polymethylmethacrylate (PMMA)	1–3	20–50
Polystyrene (PS)	1–2	30–80
Polycarbonate (PC)	2.5–3	60–70
Polyvinyl carbide (PVC)	2–3	40–50

Fracture toughness vs. strength



fracture considerations. (*Adapted from M. F. Ashby, Materials Selection in Mechanical Design, Pergamon Press, Oxford, 1992.*)