Mechanical Behaviour of Materials

Chapter 14
Fracture mechanics

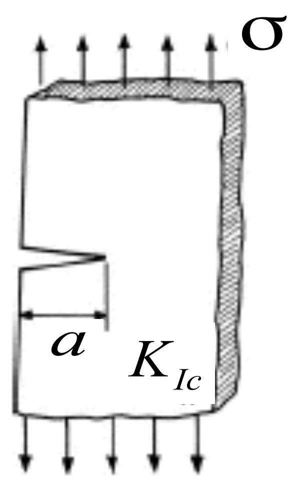
Image obtained from *THE TITANIC - SINKING - 15 April 1912*
http://www.solarnavigator.net/titanic sinking.htm

Dr.-Ing. 郭 瑞 昭

Facture Mechanics

A material fracture depends on temperature, the stress state and with time, and the environment.

Applied stress



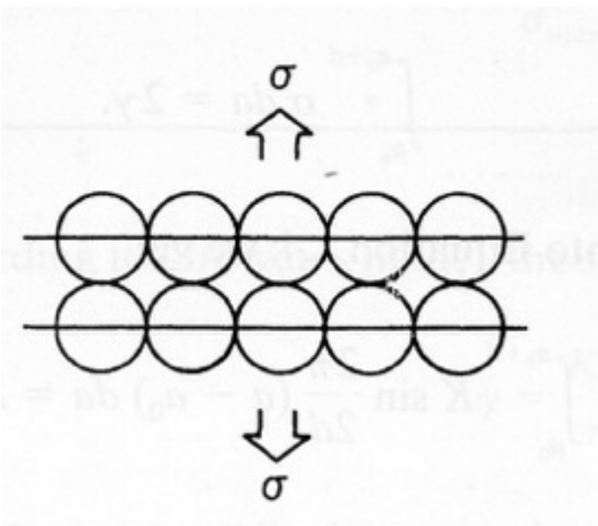
Flaw size

Fracture toughness

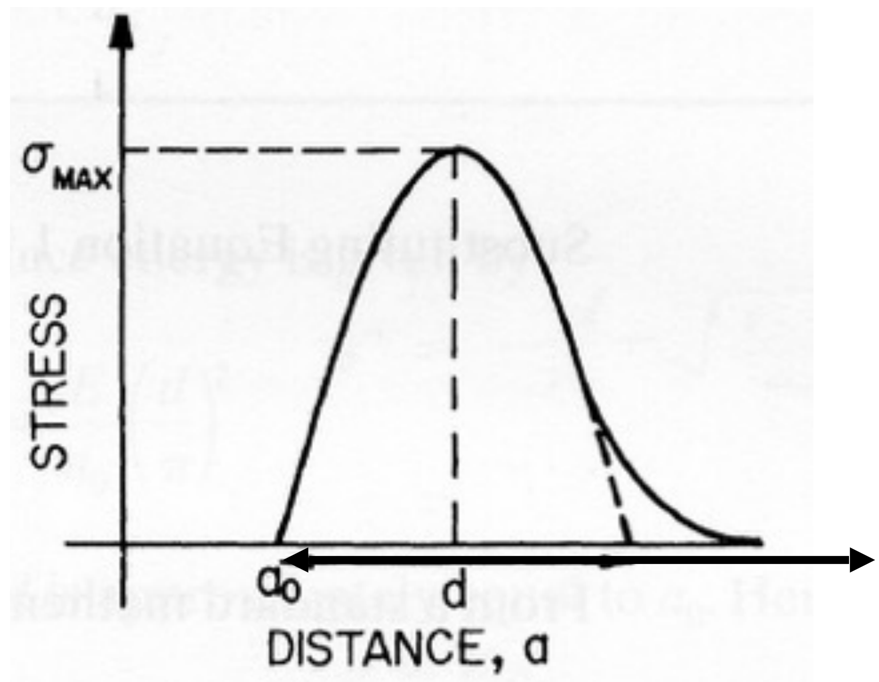
Theoretical facture strength

Under normal stress a material is to cleave, when the fracture surface is perpendicular to the applied stress.

The atoms are separated along the direction of the applied stress.



Stress required to separate two atomic layers



Theoretical facture strength: stress intensity approach

$$1.\ \sigma = \sigma_{th} \sin\left(\frac{2\pi}{d} x\right)$$

$$2.\ \frac{dx}{a_0} = d\varepsilon$$

$$3.\ \frac{d\sigma}{d\varepsilon} = \frac{d\sigma}{dx / a_0} = E$$

$$a_0 \frac{d\sigma}{dx} = \frac{2\pi}{d} \sigma_{th} \cos\left(\frac{2\pi x}{d}\right)$$

$$\frac{d\sigma}{dx} = \left(\frac{2\pi}{d} \sigma_{th}\right)_{x \to 0}$$

$$\sigma_{th} \approx \frac{E}{2\pi} \approx \frac{E}{10}$$

$$\sigma_{th} = \frac{d}{2\pi a_0} E$$

Theoretical facture strength: Energy criterion

$$\int_0^{d/2} \sigma_{th} \sin\left(\frac{2\pi x}{d}\right) dx = 2\gamma$$

$$\sigma = \sigma_{th} \sin\left(\frac{2\pi}{d} x\right)$$

$$\left(\sigma_{th}^2\right) = \frac{\gamma E}{a_0}$$

$$\sigma_{th} = \sqrt{\frac{E\gamma}{a_0}}$$

Theoretical Cleavage Strength

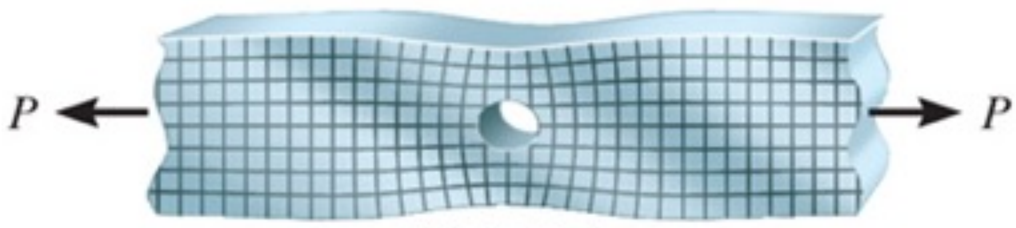
Table 7.1 Theorectical Cleavage Stresses According to Orowan's Theory*

Element	Direction	Young's Modulus (GPa)	Surface Energy (J/m2	σ_{max} (GPa)	σ_{max}/E
σ-Iron	<100>	132	2	30	0.23
	<111>	260	2	46	0.18
Silver	<111>	121	1.13	24	0.20
Gold	<111>	110	1.35	27	0.25
Copper	<111>	192	1.65	39	0.20
	<100>	67	1.65	25	0.38
Tungsten	<100>	390	3.00	86	0.22
Diamond	<111>	1,210	5.4	205	0.17

* Adapted with permission from A. Kelly, *Strong Solids*, 2nd ed. (Oxford, U.K.: Clarendon Press, 1973), p. 73.

Crack-initiated fracture: stress concentration

The fundamental requisite for the propagation of a crack is that the stress at the tip of the crack must exceed the theoretical cohesive strength of the material.



Actual stress distribution

$$K = \frac{\sigma_{max}}{\sigma_a}$$

The stress concentration factor (SCF) is as the ratio of the maximum stress to the applied stress.

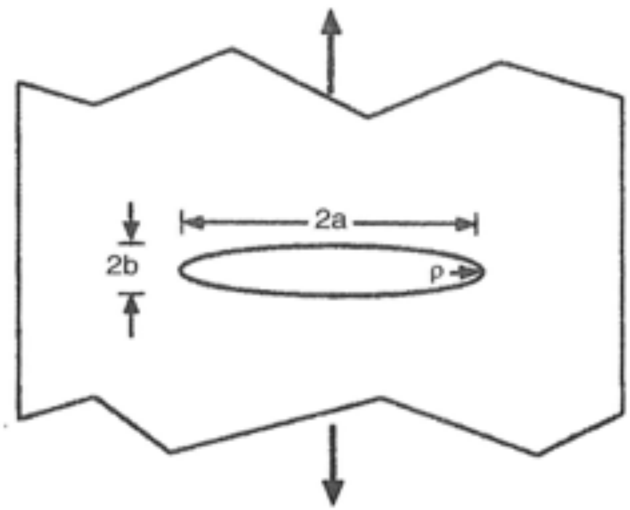
Stress concentration factor (stress approach)

Inglis: The stress rises dramatically near the hole and has a maximum value at the edge of the hole. The maximum value is given:

External applied stress

$$\sigma_{max} = \sigma_a \left(1 + 2\frac{a}{b}\right)$$

stress concentration factor



$$\sigma_{max} = \sigma_a \left(1 + 2\sqrt{\frac{a}{\rho}}\right) \cong 2\sigma_a \sqrt{\frac{a}{\rho}}$$

having a radius of curvature

$$\rho = \frac{b^2}{a}$$

$\rho << a$

$$\sigma_{max} = 2\sigma_a \sqrt{\frac{a}{\rho}}$$

$$K = 2\sqrt{\frac{a}{\rho}}$$

stress concentration factor

$$K = \frac{\sigma_{max}}{\sigma_a}$$

Stress concentration factor (stress approach)

$$\sigma_{max} = \sigma_a \left(1 + 2\frac{a}{b}\right)$$

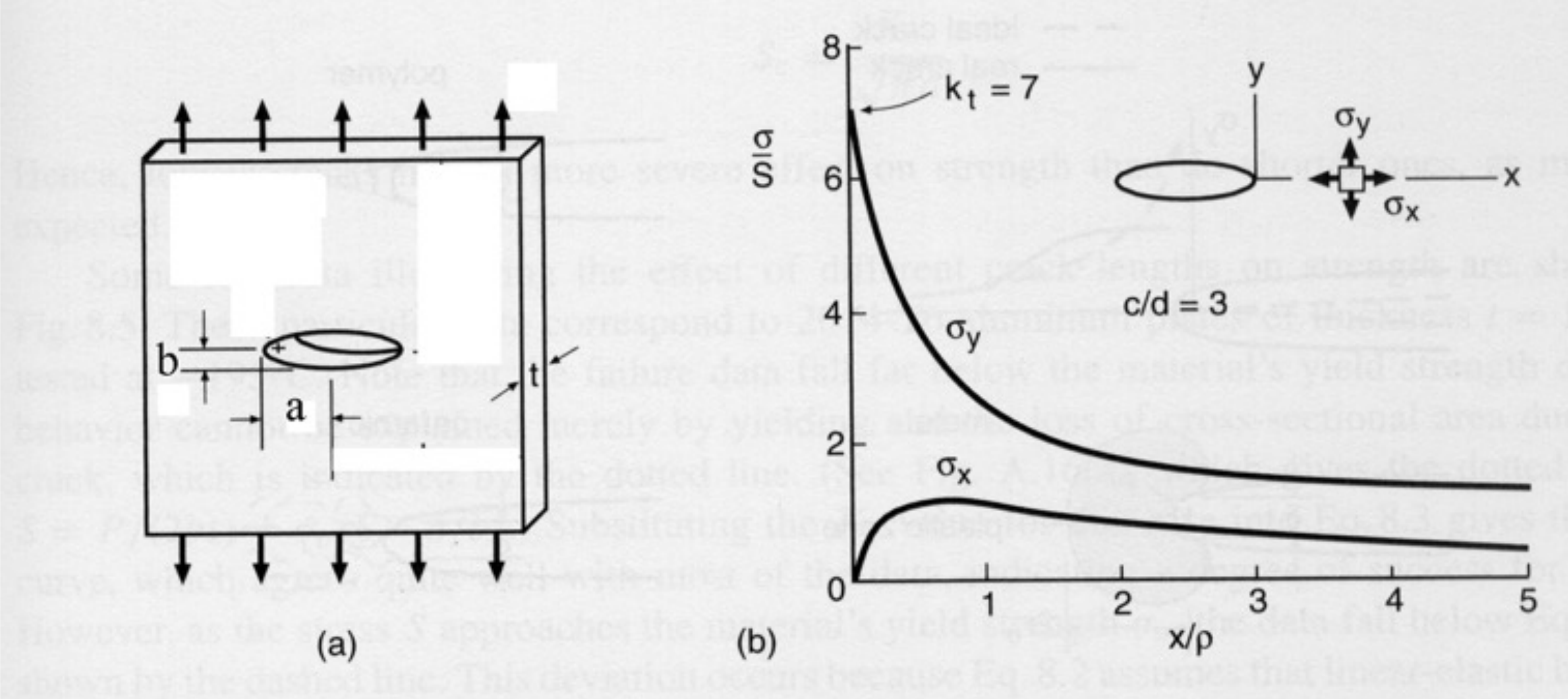
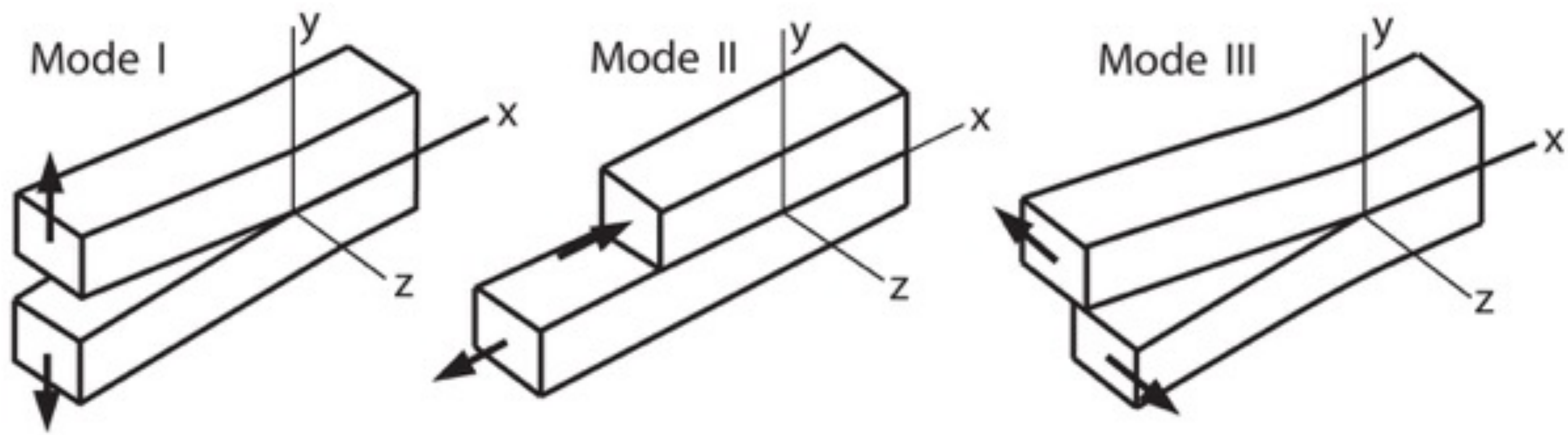


Figure 8.3 Elliptical hole in a wide plate under remote uniform tension, and the stress distribution along the *x*-axis near the hole for one particular case.

Facture modes



Shear mode

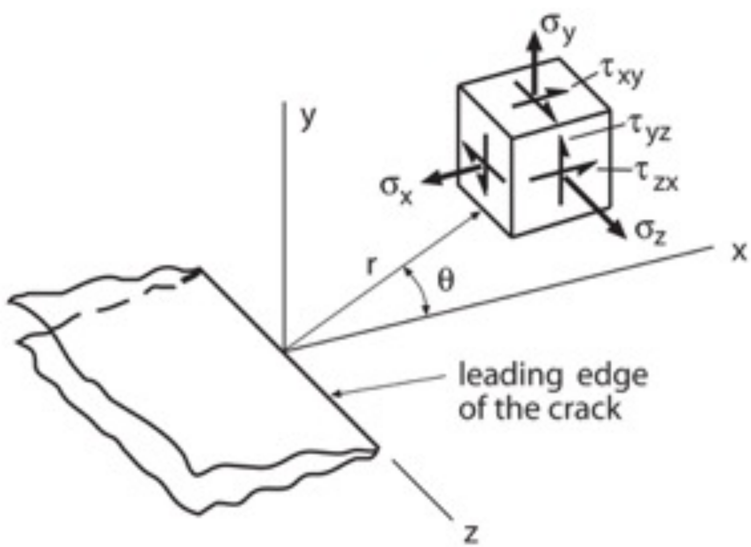
Irwin's fracture analysis: stress intensity factor

Irwin proposed the stress state around an Infinitely sharp crack in a semi-infinite elastic solid

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$



$$\sigma_z = 0$$ plane stress

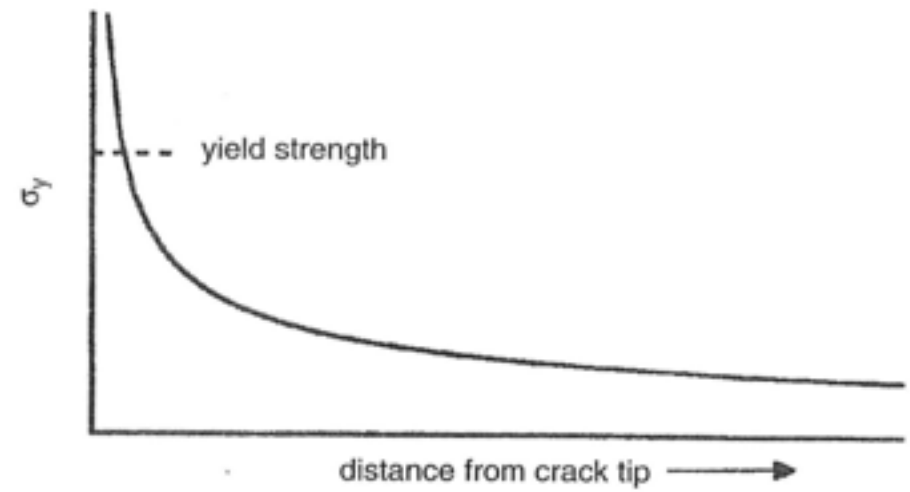
$$\sigma_z = \upsilon(\sigma_x + \sigma_y)$$ plane strain

$$\tau_{yz} = \tau_{zx} = 0$$

Stress intensity factor

Stress intensity factor in a semi-infinite body is given:

$$K_I = \sigma\sqrt{\pi a}$$



Stress intensity factor for finite body is given:

$$K_I = f\sigma\sqrt{\pi a}$$

f depends on the specimen geometry
and is >1 for small crack

Fracture occurs when K_I reaches a critical value, K_{Ic}, which is a material property.

$$\sigma_f = \frac{K_{Ic}}{f(\pi a)^{1/2}} \qquad f = 1 \qquad K_{Ic} = \sqrt{EG_c}$$

Comparison between K and K_{IC}

K_I (the stress intensity factor): provides a complete description of the state of stress, strain and displacement over some region of the body, is dependent of the crack length and the geometry of the body.

$$K_I = \sigma \cdot \sqrt{\pi a} \cdot f\left(\frac{a}{w}\right)$$

K (the stress concentration factor): determines the magnitude of the stress at a single point, is independent of the crack length.

$$K = \frac{\sigma_{\max}}{\sigma_a}$$

Griffith theory (energy criterion)

$$\Delta U_{surf} = 4\,a \cdot t \cdot \gamma$$

thickness

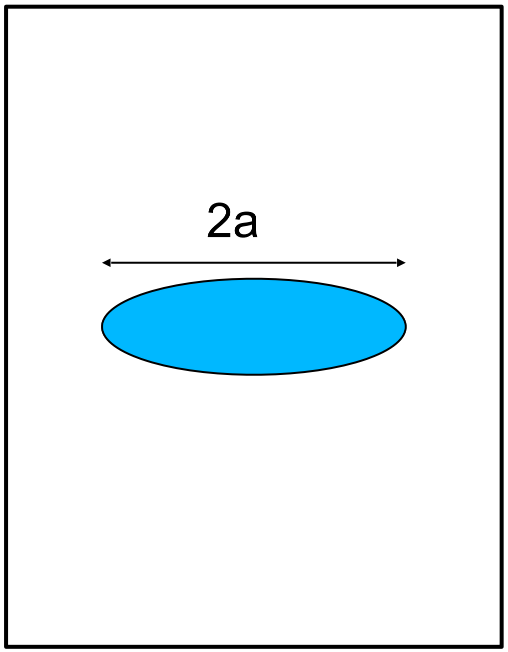
$$\Delta U_{elast} = \left(\frac{\sigma^2}{2E}\right)\left(2\pi a^2 t\right) = \frac{\pi\, a^2 t \sigma^2}{E}$$

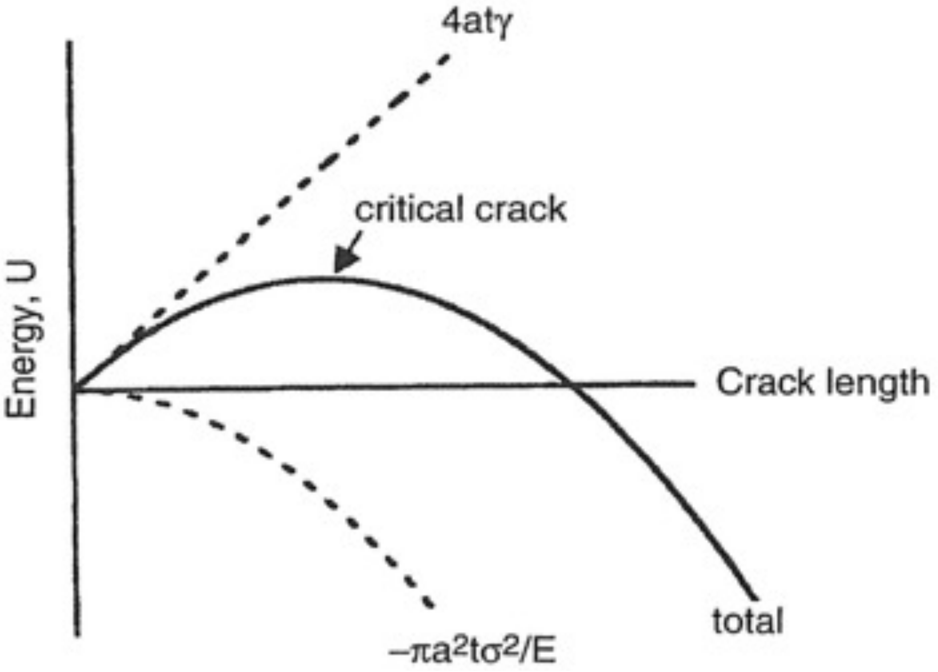
$$\Delta U_{total} = 4at\gamma - \frac{\pi\, a^2 t \sigma^2}{E}$$

$$\frac{d\Delta U_{total}}{da} = 4t\gamma - \frac{2\pi\, a^2 t \sigma^2}{E} = 0$$

Fracture stress

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi a}}$$

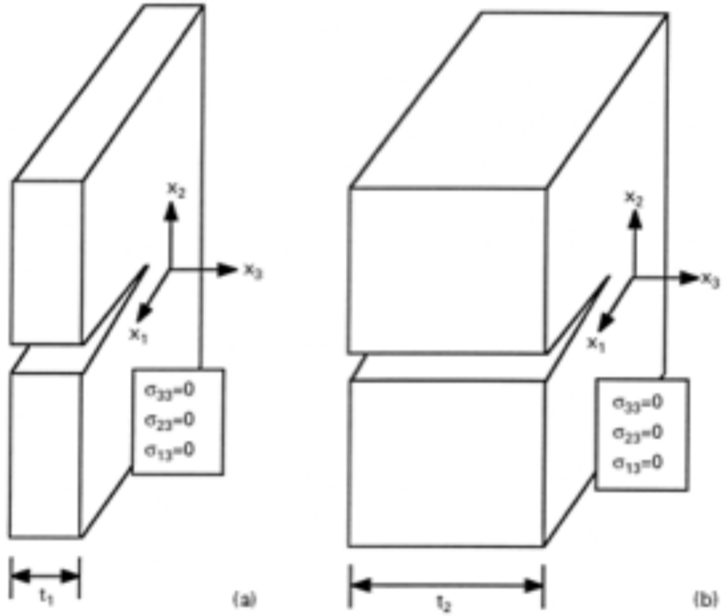




Griffith theory-conti.

Plane stress Plane strain

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}}$$



$$\sigma_f = \sqrt{\frac{2E\gamma}{(1-\nu^2)\pi a}}$$

Crack in (a) thin (t_1) and (b) thick (t_2) plates. Note the plane-stress state in (a) and the plane-strain state in (b).

Fracture toughness

TABLE 7.1 Typical Ranges of Plane Strain Fracture Toughness and Yield Strength for Several Materials at Room Temperature

Material	K_{Ic} (MPa$\sqrt{m}$)	Y (MPa)
Al 2000 series	24–40	300–450
Al 7000 series	25–35	400–600
Ti-6A1-4V alloys	50–110	800–1100
4340 steel	55–105	1300–1700
Maraging steels	40–80	1400–2300
Alumina (Al_2O_3)	3–5	—
Boron carbide (BC)	4–6	—
Silicon nitride (Si_3N_4)	4–8	—
Silicon carbide (SiC)	2–5	—
Tetragonal zirconias (doped ZrO_2)	4–10	—
Epoxies	0.5–0.8	—
Borosilicate glass	0.5–1	
Polymethylmethacrylate (PMMA)	1–3	20–50
Polystyrene (PS)	1–2	30–80
Polycarbonate (PC)	2.5–3	60–70
Polyvinyl carbide (PVC)	2–3	40–50

Orowan theory: energy including plastic energy

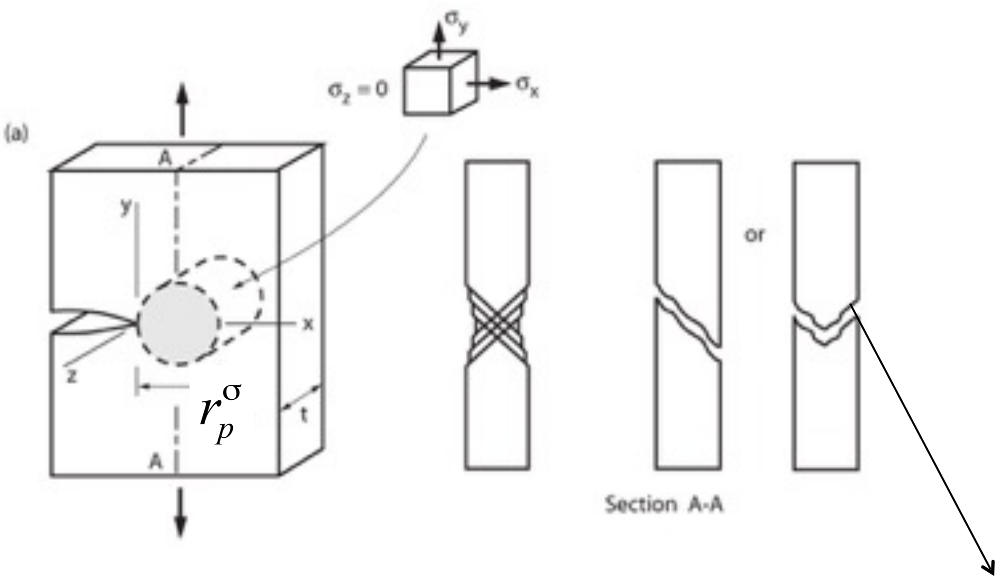
$$\sigma_f = \frac{K_c}{\sqrt{\pi a}}$$

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}} \longrightarrow \sigma = \sqrt{\frac{EG_c}{\pi a}}$$

$$G_c = 2\gamma + E_p$$ Including the plastic work in generating the fracture surface

$$K_c = \sqrt{EG_{Ic}} = \sigma_c\sqrt{\pi a}$$ Fracture toughness

Shear fracture: Plane stress



$$\sigma_x \neq 0$$

$$\sigma_y \neq 0$$

$$\sigma_z = 0$$

Slant-type fracture

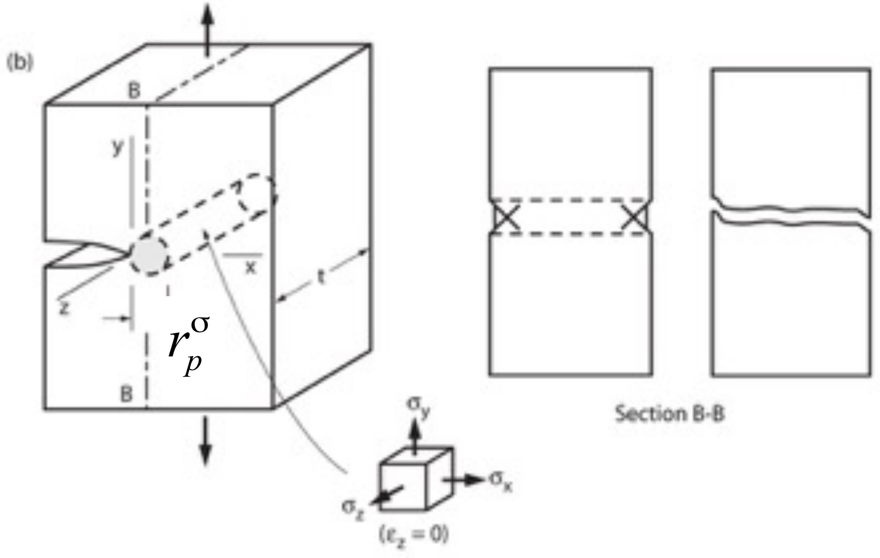
Remarks:

1.The toughness of very thin sheets is quite low, that is, the type of low-energy fracture.

2. Plane stress toughness is not a material property.

3. Biaxial stress state

Flat fracture: Plane strain



$$\varepsilon_x \neq 0 \qquad \sigma_x \neq 0$$

$$\varepsilon_y \neq 0 \qquad \sigma_y \neq 0$$

$$\varepsilon_z = 0 \qquad \sigma_z = \nu\left(\sigma_x + \sigma_y\right)$$

Remarks:

1. The triaxial stress state of plane strain reduces the plastic zone size in comparison to the plane stress zone size.
2. The triaxial stress state is pronounced at the boundary between the plastic and elastic zones.

Plastic zone size

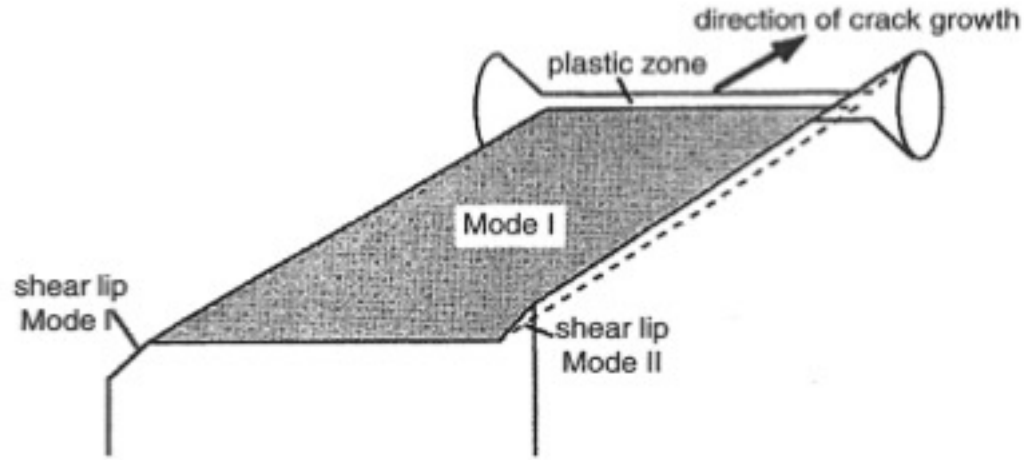


Figure 14.8. A three-dimensional sketch showing the shape of the plastic zone and the shear lip formed at the edge of the plate.

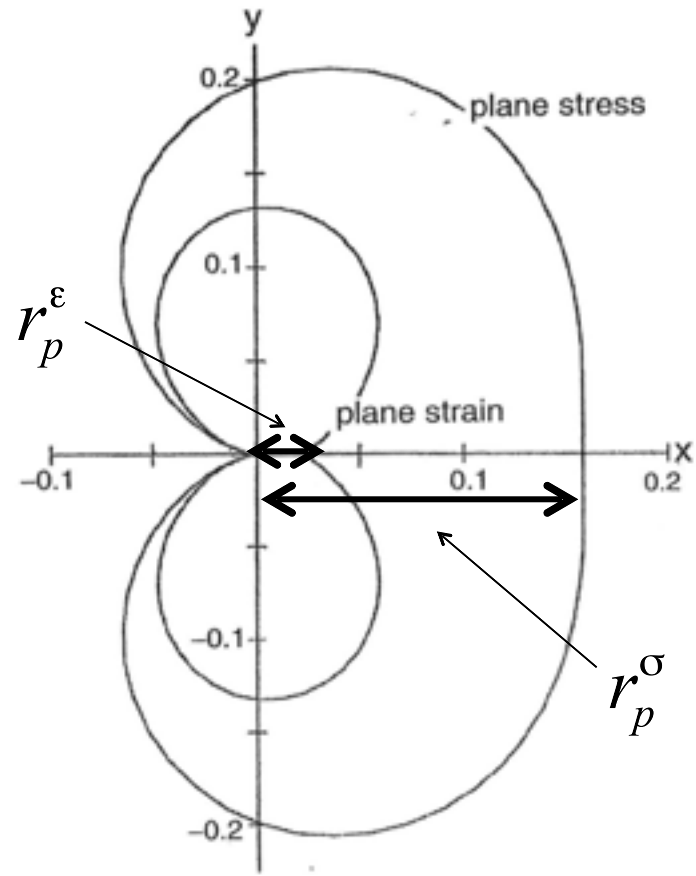


Figure 14.9. Plots of the plastic zones associated with plane stress and plane strain. Dimensions are in units of $(K_I/Y)^2$. The curve for plane strain was calculated for $\upsilon = 0.3$. Note that the "radii" of the plane-stress and plane-strain zones are conventionally taken as $(K_I/Y)^2/(2\pi) = 0.159$ and $(K_I/Y)^2/(6\pi) = 0.053$, respectively.

Plane stress

$$r_p = \frac{\left(\frac{K_{Ic}}{Y}\right)^2}{2\pi}$$

Plane strain

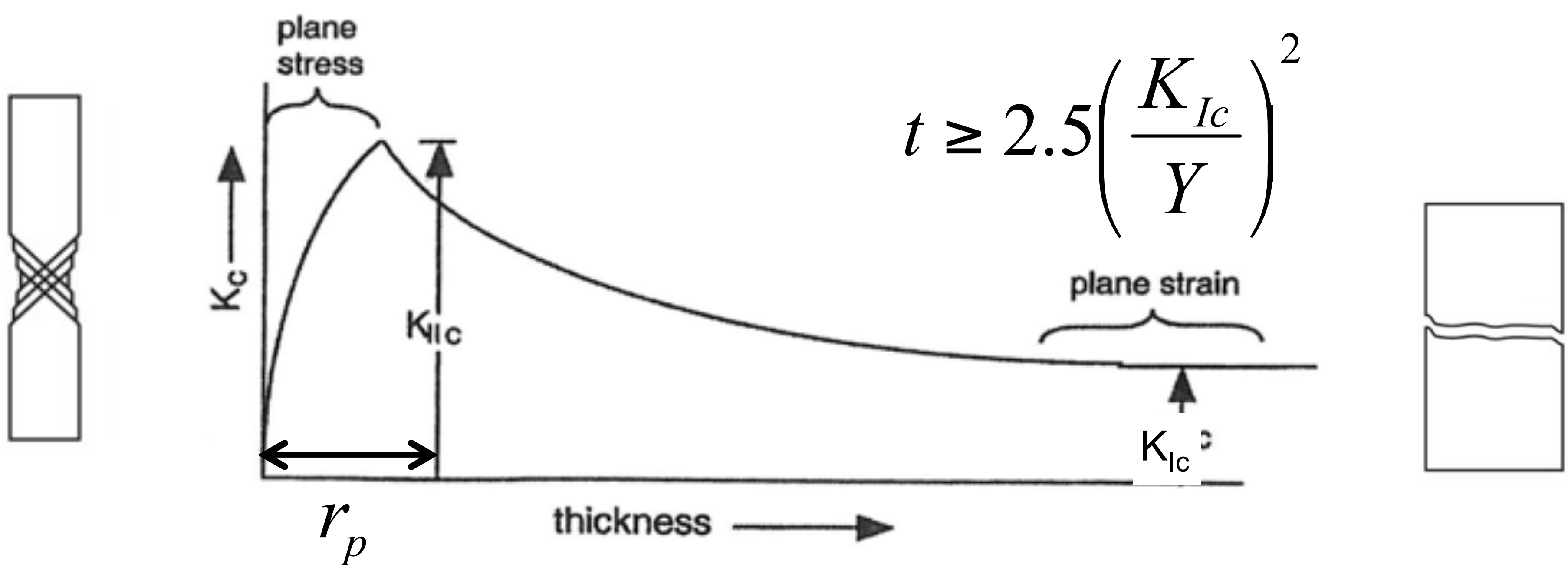
$$r_p = \frac{\left(\frac{K_{Ic}}{Y}\right)^2}{6\pi}$$

Effect of thickness on K_c

The thickness of the specimen should be much greater than the radius of the plastic zone for plane stress:

$$t \le 2 \cdot r_p$$

$$t \ge 2.5\left(\frac{K_{Ic}}{Y}\right)^2$$



Fracture toughness testing

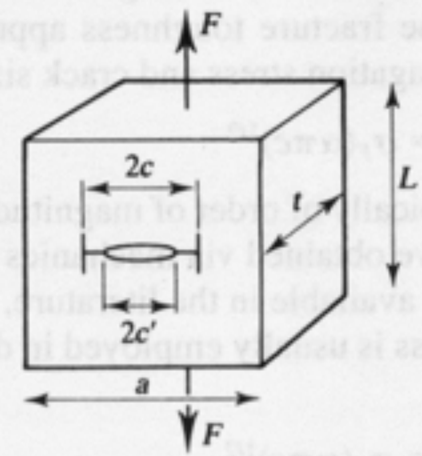
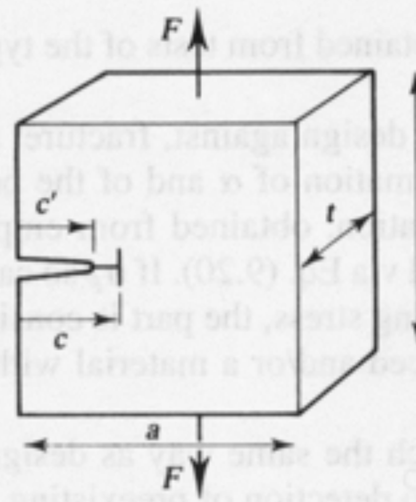
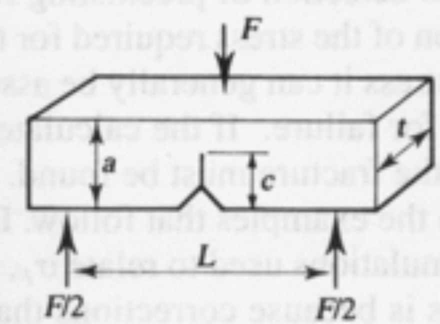
The following are the fracture toughness parameters commonly obtained from testing

- **K (stress intensity factor)** can be considered as a **stress**-based estimate of fracture toughness. K depends on geometry (the flaw depth, together with a geometric function, which is given in test standards for each test specimen geometry).

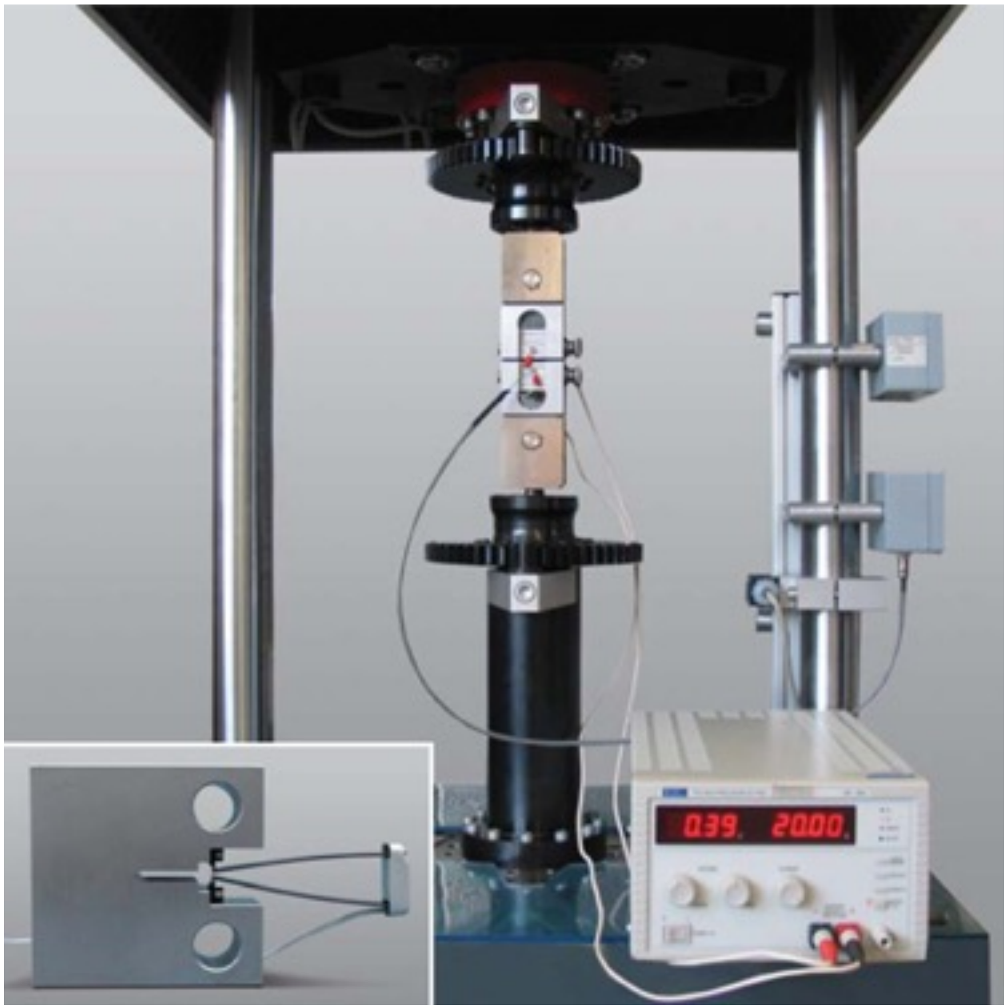
- **CTOD (crack-tip opening displacement)** can be considered as a **strain**-based estimate of fracture toughness. However, it can be separated into elastic and plastic components. The elastic part of CTOD is derived from the stress intensity factor, K. The plastic component is derived from the crack mouth opening displacement (measured using a clip gauge).

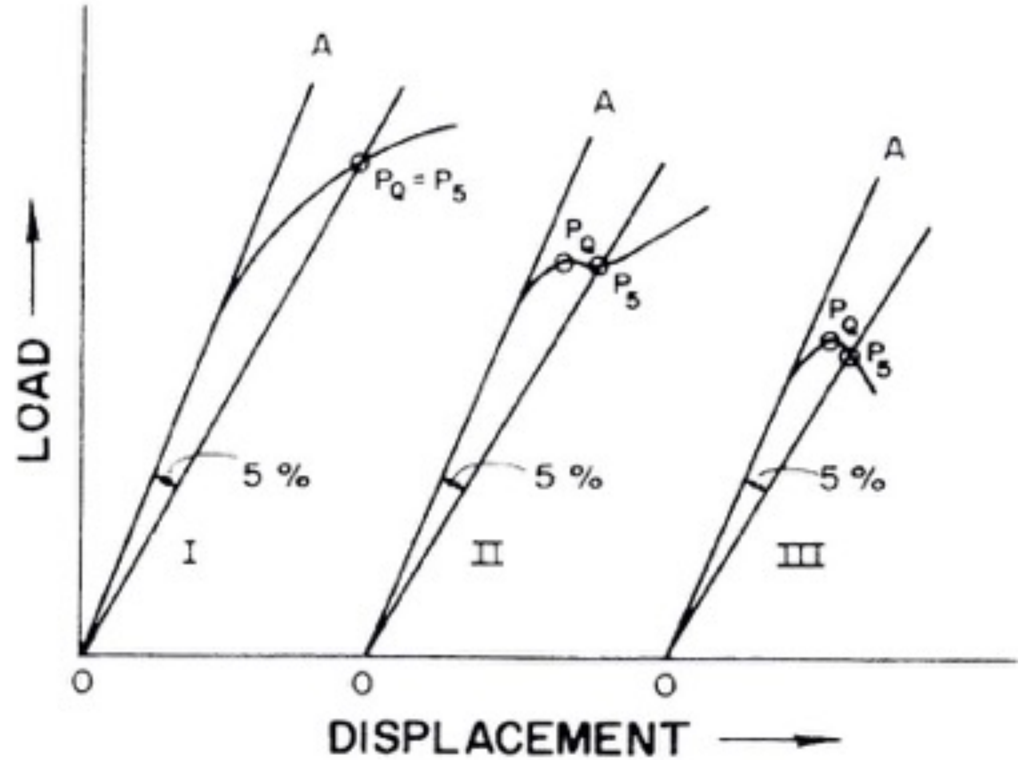
- **J (J-integral)** is an **energy**-based estimate of fracture toughness. It can be separated into elastic and plastic components. As with CTOD, the elastic component is based on K, while the plastic component is derived from the plastic area under the force-displacement curve.

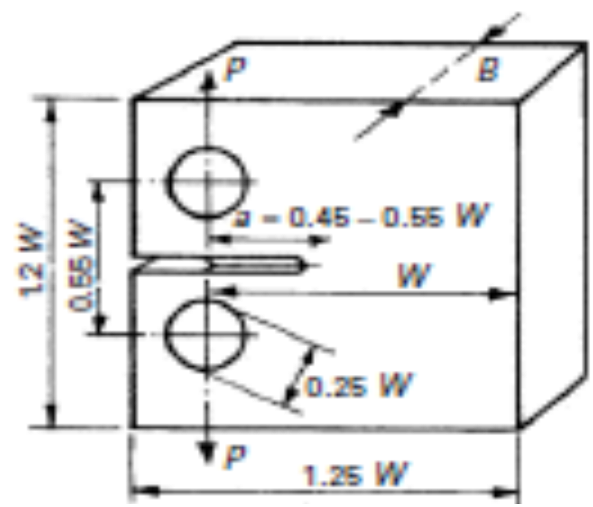
Determination of fracture toughness

Specimen type	$f(c/a)$	Comments
Center notched tension $K_{Ic} = \sigma_F \sqrt{\pi c}\, f(c/a)$ $\sigma_F = F_c/ta$ 	$\left(\frac{a}{\pi c} \tan \frac{\pi c}{a}\right)^{1/2}$	Specifications $L = 4a$, $2c = a/3$ For K_{Ic} $10 > a/t > 5$
Compact tension $K_{Ic} = \sigma_F \sqrt{\pi c}\, f(c/a)$ $\sigma_F = F_c/ta$ 	$\frac{1}{\sqrt{\pi}}\left[29.6 - 185.5\left(\frac{c}{a}\right) + 655.7\left(\frac{c}{a}\right)^2 - 1017\left(\frac{c}{a}\right)^3 + 638.9\left(\frac{c}{a}\right)^4\right]$	Specifications $c = a/2$ For K_{Ic} $a/t > 2$
Three-point bend $K_{Ic} = \sigma_F \sqrt{\pi c}\, f\left(\frac{c}{a}\right)$ $\sigma_F = F_c/ta$ 	$\frac{L}{\sqrt{\pi a}}\left[2.9 - 4.6\left(\frac{c}{a}\right) + 21.8\left(\frac{c}{a}\right)^2 - 37.6\left(\frac{c}{a}\right)^3 + 387\left(\frac{c}{a}\right)^4\right]$	Specifications $L = 8a$, $c = a/5$ For K_{Ic} $8 > a/t > 2$

Measurement of fracture toughness







Fracture toughness: Charpy test

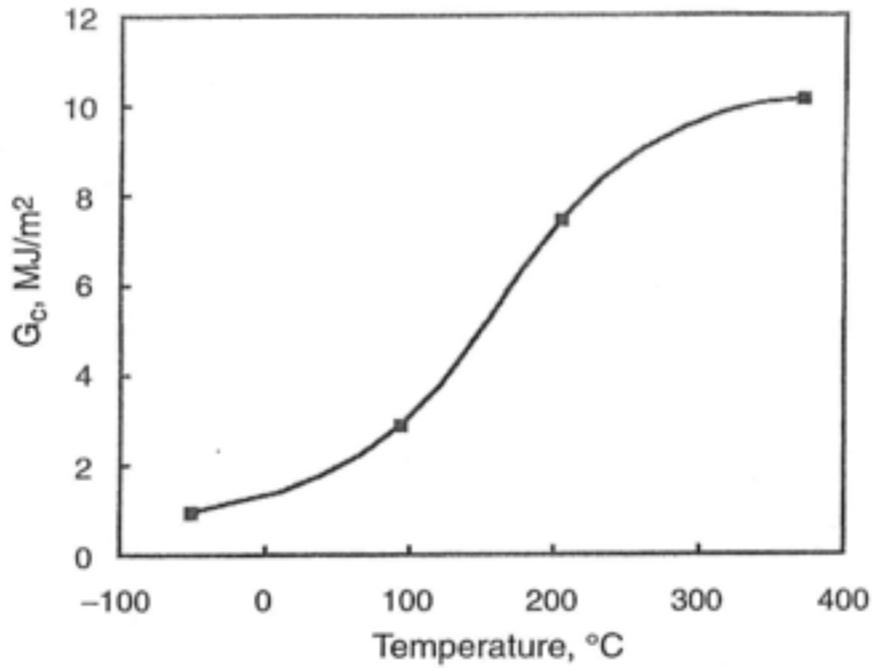


Figure 14.13. The dependence of G_c on temperature for steel forgings. Data from D. H. Winne and B. M. Wundt, *Trans. ASME*, Vol. 80 (1958).

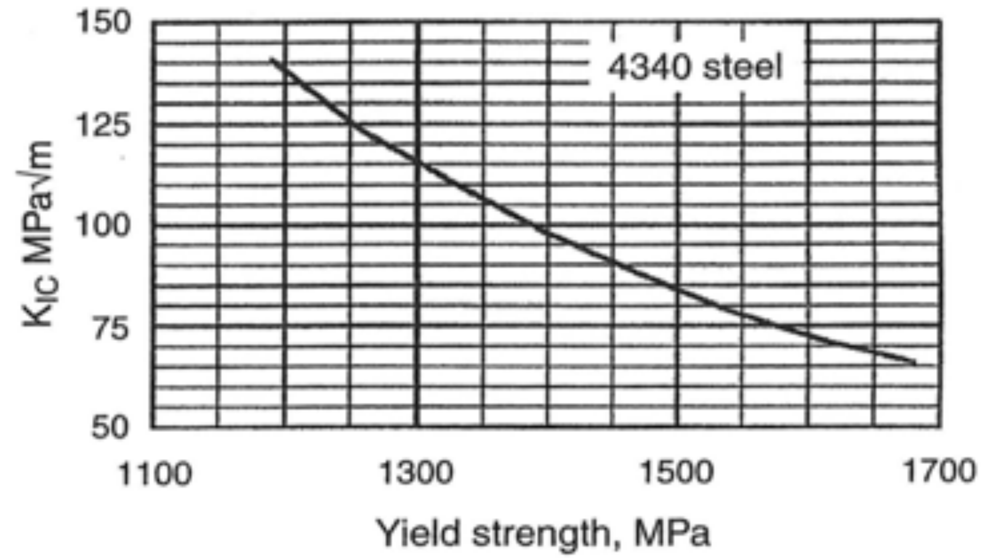
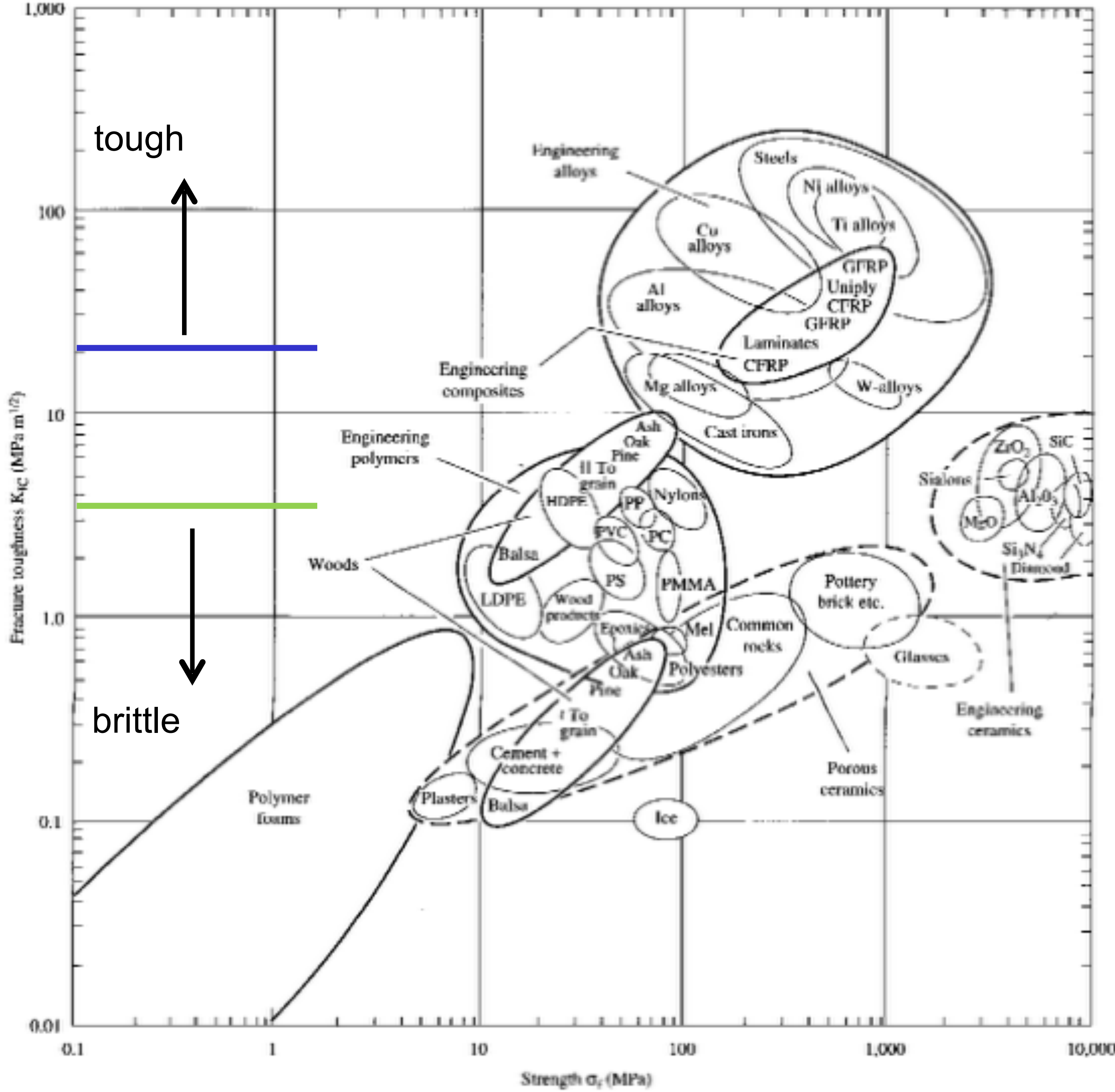


Figure 14.14. The inverse correlation of K_{Ic} with yield strength for 4340 steel.

An increased loading rate has an similar effect to decreased temperature

Fracture toughness vs. strength



fracture considerations. (*Adapted from M. F. Ashby, Materials Selection in Mechanical Design, Pergamon Press, Oxford, 1992.*)