Engineering Mechanics of Materials

Chapter 01

Tension, Compression, and Shear

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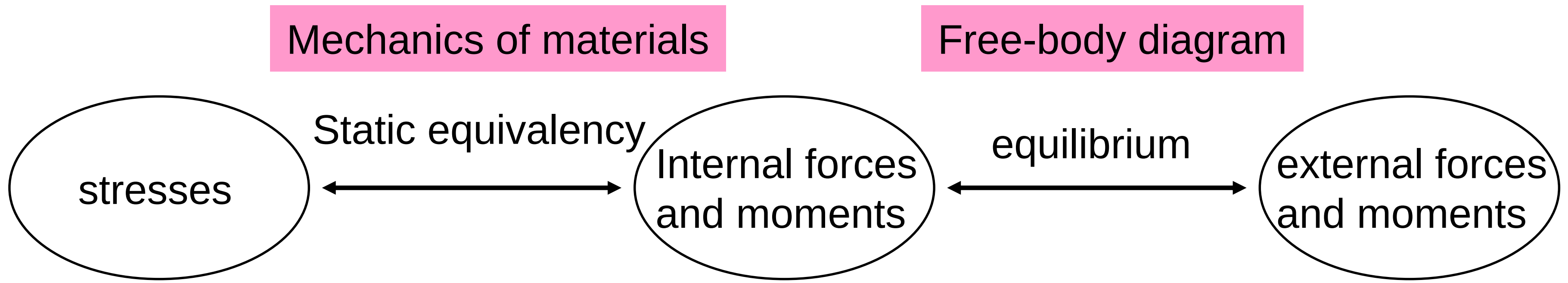
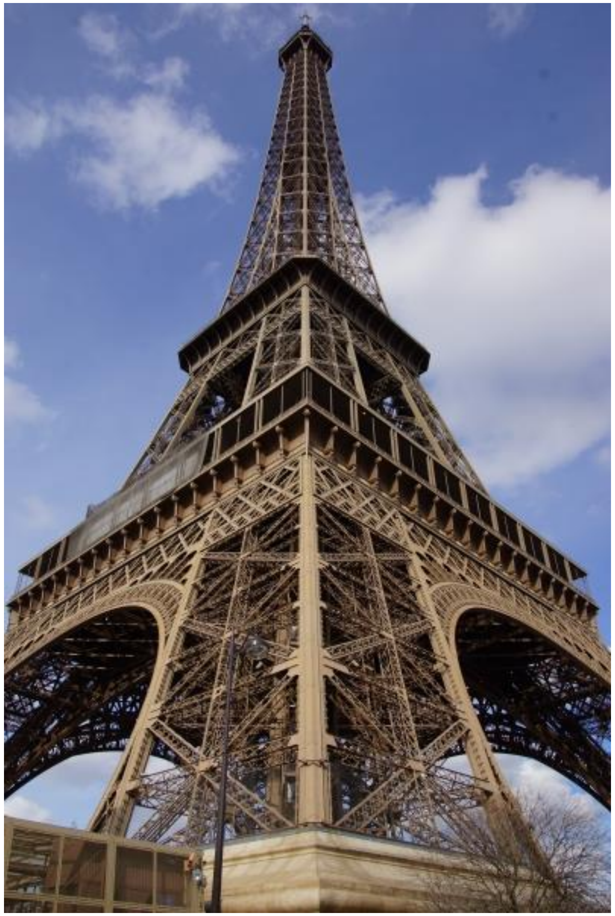
Introduction



Learning objectives

- Understanding the concept of stress.
- Understanding the two step analysis of relating stresses to external forces and moments.

Why we learn mechanics of materials?



Learning Objectives

- Define “mechanics of materials”.
- Study normal stress (σ) and normal strain (ε).
- Identify key properties of various materials:
  - Modulus of elasticity (E), yield stress (σy) and ultimate stress (σu)
- Plot shear stress (τ) vs. shear strain (γ).
- Identify the shearing modulus of elasticity (G).
- Study Hooke’s law for normal and shear stresses and strains.
- Investigate changes in lateral dimensions and volume of a bar.
- Study normal, shear, and bearing stresses in bolted connections.
- Use factors of safety to establish allowable values of stresses.
- Introduce basic concepts of design.

Introduction to Mechanics of Materials

- "Mechanics of Materials"– a branch of applied mechanics that deals with the behavior of solid bodies subjected to various types of loading.
  - AKA "Strength of Materials" and "Mechanics of Deformable Bodies"

- Principal Objective– determine the stresses, strains, and displacements in structures and their components due to loads acting on them.

- Analytical methodologies will be applied to real bodies– that is, bodies of finite dimensions that deform under loads.

Problem-Solving Approach

- Step One: Conceptualize [Hypothesize, Sketch]
  - List all relevant data provided within the problem statement.
  - Draw a sketch showing all applied forces, boundary conditions, and interactions between adjacent bodies.
- Step Two: Categorize [Simplify, Identify]
  - Identify all unknowns within the problem.
  - Make any necessary assumptions to simplify the problem.
- Step Three: Analyze [Evaluate, Select Equations, Apply Mathematics]
  - Apply appropriate theory and equations and solve for the unknowns.
- Step Four: Finalize [Conclude, Examine Answer– Sensible? Appropriate Units?]
  - Compare answer to similar problem solutions, scrutinize the units.
  - Vary key parameters to test the robustness of your solution.

Equilibrium Equations

- Vector Form

Resultant Force R

$$R = \sum F = 0$$

Resultant Moment M

$$M = \sum M = \sum (r \times F) = 0$$

- Scalar Form

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

Applied Forces

- Concentrated Forces/Moments– applied to a single point.
- Distributed Forces– applied along a portion of the member’s length.
- Free-Body Diagram– includes all applied forces and moments, reaction forces and moments, and connection forces between individual components.
- Static Sign Convention
  - Forces acting in the positive direction of the coordinate axes are positive.
  - The right-hand-rule is used to determine the signs of moment vectors.

Distributed Forces

Distributed forces can be broken down into surface forces and body forces. Surface forces are distributed forces where the point of application is an area (a surface on the body). Body forces are forces where the point of application is a volume (the force is exerted on all molecules throughout the body). Below are some examples of surface and body forces.

Reactive Forces and Support Conditions

- Forces at structural supports which usually result from the action of applied forces to the overall structure.
- Reaction Force– shown as a single arrow with a slash drawn through it.
- Reaction Moment– shown as a double-headed arrow or curve with a slash.
- Table 1-1 lists a wide range of support or connection types along with their corresponding sketch and reaction forces (see page 8 of the text).

Internal Forces (Stress Resultants)

- The internal axial force, torsional moment, transverse shear, and bending moment present within the member.

- A sectional cut normal to the axis of each member allows for internal forces to be represented within the FBD.

- Deformation Sign Convention
  - Tension is positive
  - Compression is negative

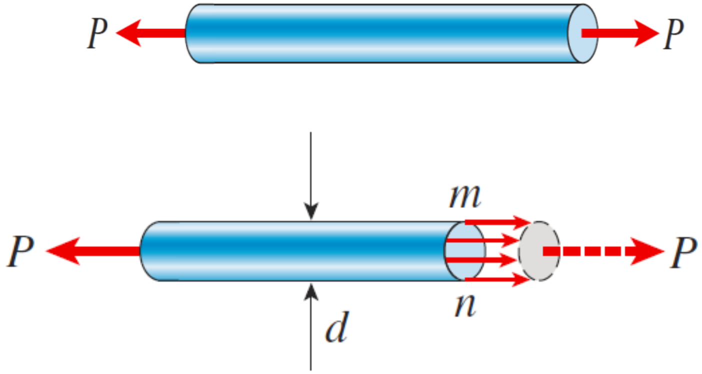
Normal Stress and Strain

Normal Stress

- Symbol: σ (sigma)
- Equation:

$$\sigma = \frac{P}{A}$$

- Units: force per unit of area
  - IP:
    - Pounds per square inch (psi)
    - Kips per square inch (ksi)
  - SI:
    - Newtons per square meter (N/m2)
    - Pascals (Pa)



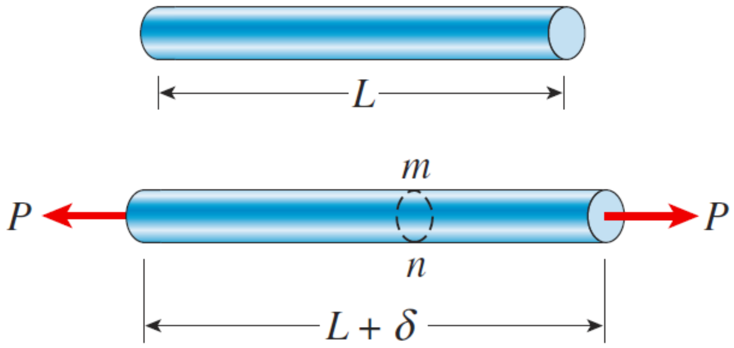
Normal Stress and Strain

Normal Strain

- Symbol: ε (epsilon)
- Equation:

$$\varepsilon = \frac{\delta}{L}$$

- Units: none (dimensionless)



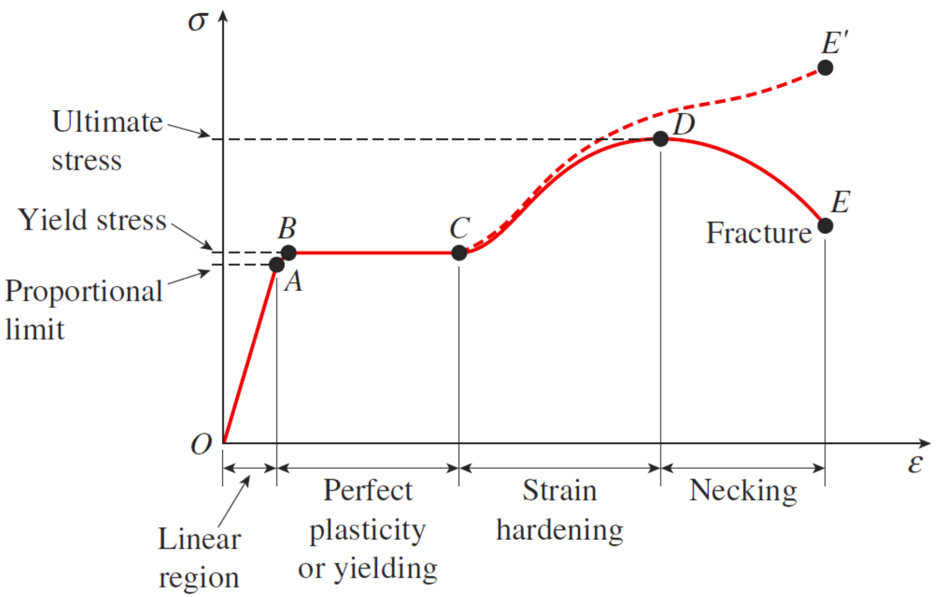
Requirements for Normal Stress and Strain

- The bar is prismatic.
- The loads act through the centroids of the cross section.
- The material is homogeneous.

Stress-Strain Diagrams

- Tension or compression tests are performed on a specimen with the stress and strain being determined at various magnitudes of loading.

- The stress is plotted against strain to achieve a stress-strain diagram.

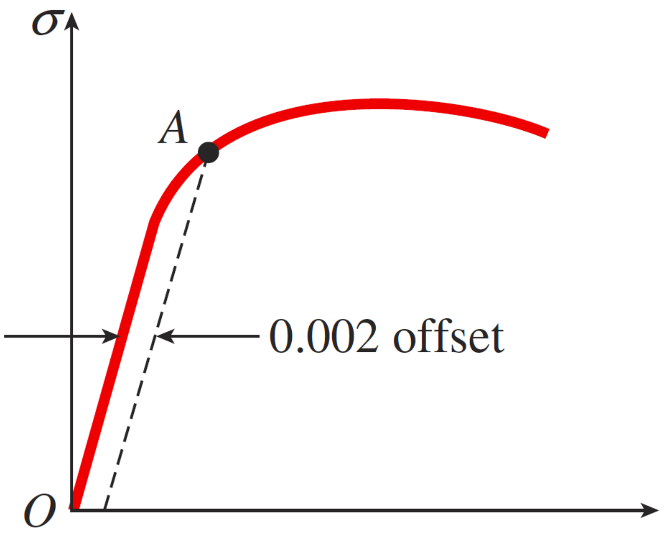


Mechanical Properties of Materials

- Proportional Limit– The point in which the stress and strain are no longer proportional to one another.
- Modulus of Elasticity (E)– The slope of the straight line the plotting of stress and strain produces within the linear region of the diagram (before proportional limit is reached).
- Yield Stress (σ_y)– The corresponding magnitude of stress at which point the material “yields” under the loading and allows for rapid increase of stress.
- Ultimate Stress (σ_u)– The corresponding magnitude of stress at which point the material begins to “neck,” thereby allowing a decrease in cross-sectional area.

Offset Method

- Used to determine an arbitrary yield stress.
- A straight line is drawn on the stress-strain diagram parallel to the initial linear part of the curve at an offset of 0.002 (or 0.2%) strain.
- The intersection of the offset line and the stress-strain curve (point A) defines the "offset yield stress".

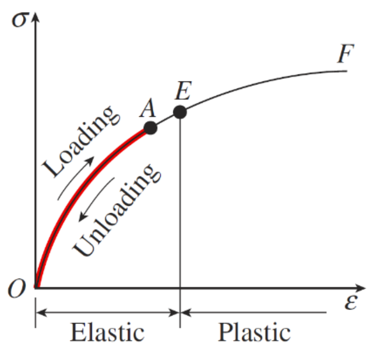


Ductility

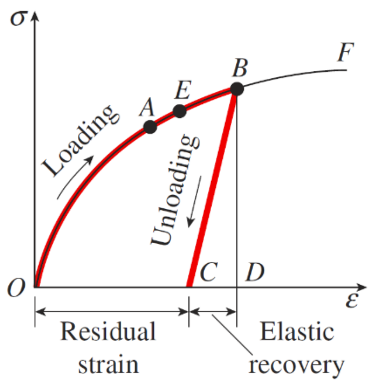
- A measure of the amount of permanent strain the material undergoes before failure occurs.
- Can be quantified by “percent elongation” and “percent reduction in area”.
- Percent Elongation: $\% \text{ Elongation} = \dfrac{L_1 - L_o}{L_o}(100)$
- Percent Reduction in Area: $\% \text{ Reduction} = \dfrac{A_o - A_1}{A_o}(100)$

Elasticity and Plasticity

- Elasticity– The property of a material by which it returns to its original dimensions during unloading.



- Partially Elastic– During unloading, the bar only returns partially to its original shape, thereby undergoing some "elastic recovery" but with "residual strain" remaining.



- Plasticity– The characteristic of a material that undergoes inelastic strains beyond the strain at the elastic limit.

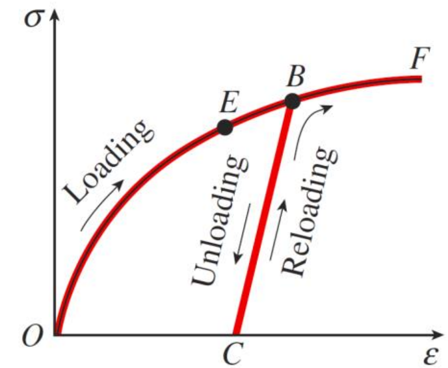
Elasticity, Plasticity, and Creep

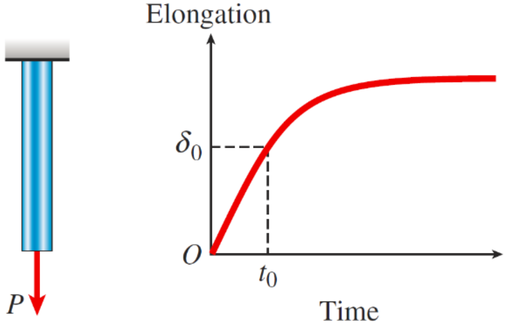
Reloading

- In elastic range – Can be loaded, unloaded, and reloaded without much change in behavior.

- In plastic range – When initially loaded and unloaded, residual strain is introduced and the properties of the material are changed (the linearly elastic region, proportional limit, and elastic limit are all increased).

- Creep– When the load is sustained for long periods of time, strains begin to develop (or "creep").





Linear Elasticity and Hooke's Law

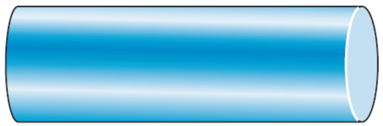
- Linear Elastic– When a material exhibits a linear relationship between stress and strain. Typically engineers design structures and machines to function within this region to avoid permanent deformations due to yielding.

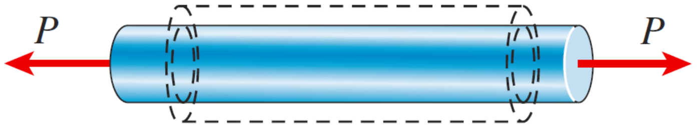
Hooke's Law

- Expression for the linear relationship between stress and strain.
  - AKA Young's Modulus
- Equation: $\sigma = E\varepsilon$
- Limitations– Relates only to the longitudinal stresses and strains in simple tension or compression of a bar (i.e. uniaxial stresses).

Poisson's Ratio

- Lateral contraction accompanies axial elongation and Poisson's Ratio relates these two strains to one another.
- Symbol: ν (nu)
- Equation: $$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon'}{\varepsilon}$$
- Units: none (dimensionless)
- Limitations:
  - The material must be homogeneous.
  - Elastic properties must be the same in all directions perpendicular to the longitudinal axis.



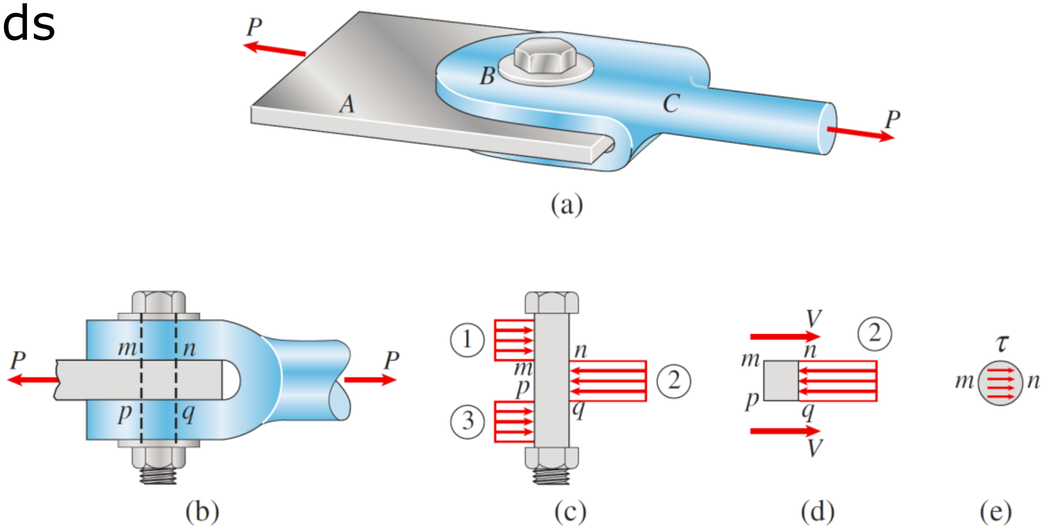


Bearing Stress

- Contact stresses which develop under tensile loads between connections.

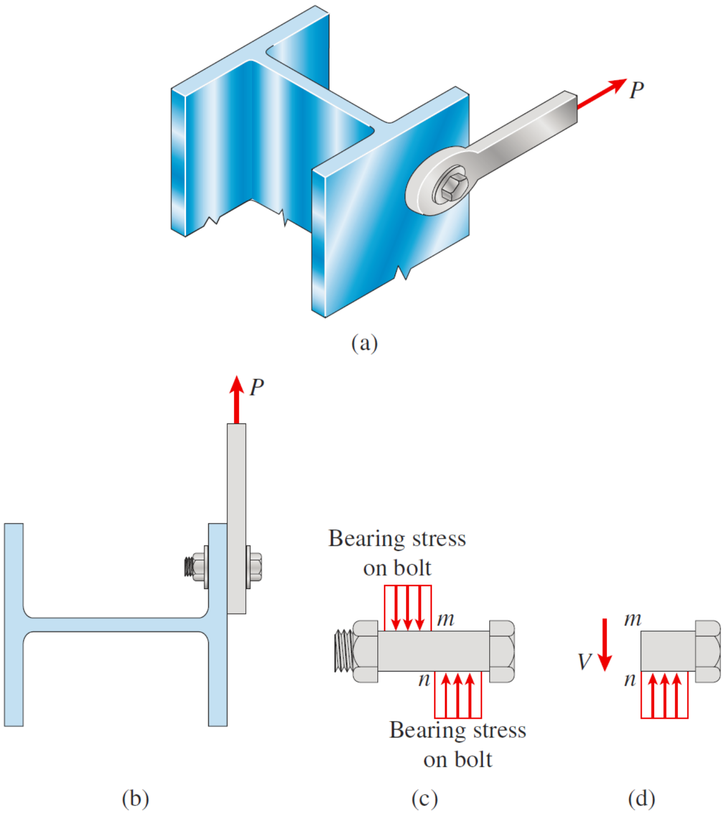
- Equation: $$\sigma_b = \frac{F_b}{A_b}$$

- Note: This calculates the "average" bearing stress.



Shear Stress

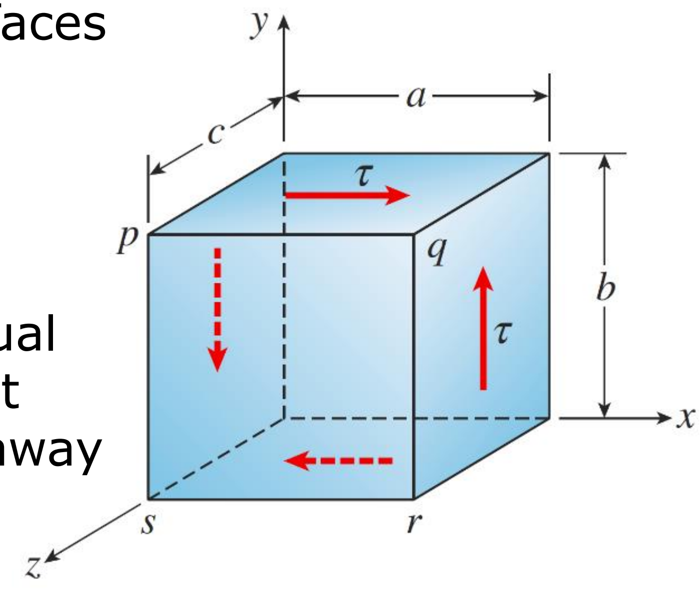
- Stress that acts tangential to the surface of the material.
- Symbol: τ (Tau)
- Equation: $$\tau_{ave} = \frac{V}{A}$$
- Units: Same as normal stress (Pa, psi, etc.)



Shear Stresses on Perpendicular Planes

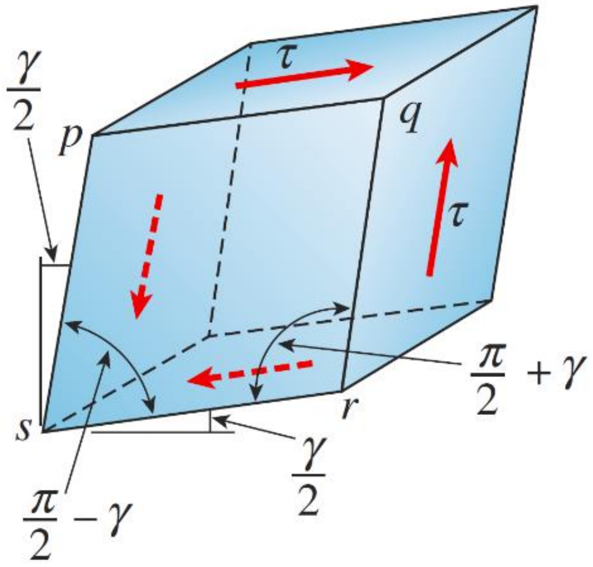
- Shear stresses on opposite (and parallel) faces of an element are equal in magnitude and opposite in direction.

- Shear stresses on adjacent (and perpendicular) faces of an element are equal in magnitude and have directions such that both stresses point toward, or both point away from, the line of intersection of the faces.



Shear Strain

- A measure of the "distortion" (or, the change in shape) of the element.
- Symbol: γ (gamma)
- Units: Degrees or Radians
- Note: The lengths of the sides of the element do not change under shear stress. Instead, the element changes in shape only.



Sign Conventions

Shear Stress

- Positive Face:
  - Positive if shear stress acts in the positive direction of a coordinate axis.
  - Negative if shear stress acts in the negative direction of a coordinate axis.
- Negative Face:
  - Positive if shear stress acts in a negative direction of a coordinate axis.
  - Negative if shear stress acts in a positive direction of a coordinate axis.

Shear Strain

- Positive when the angle between 2 positive (or 2 negative) faces is reduced.
- Negative when the angle between 2 positive (or 2 negative) faces is increased.

Shear Stress and Strain

Hooke's Law in Shear: $\tau = G\gamma$

Shear Modulus of Elasticity (G): $$G = \frac{E}{2(1+\nu)}$$

- AKA Modulus of Rigidity
- Units are same as the elastic modulus (i.e. Pa, psi, etc.)

Allowable Stresses and Allowable Loads

- Strength– The capacity of the object to support or transmit loads.

Factor of Safety

- For successful structures, the actual strength must exceed the required strength.
- Equation:

$$\text{Factor of Safety } n = \frac{\text{Actual Strength}}{\text{Required Strength}}$$

- To avoid failure, Factor of Safety > 1

- Margin of Safety: $\text{Margin of Safety} = (\text{Factor of Safety } n) - 1$

Allowable Stresses

- AKA working stress
- Equation:

$$\text{Allowable Stress} = \frac{\text{Yield Strength}}{\text{Factor of Safety } n}$$

<u>Tension</u>

$$\sigma_{allow} = \frac{\sigma_{yield}}{n}$$

<u>Shear</u>

$$\tau_{allow} = \frac{\tau_{yield}}{n}$$

- Yielding begins when the yield stress is reached at **any** point within the structure.
- For brittle materials, the ultimate stress can be used instead of the yield stress.

Allowable Loads

- Equation: $\text{Allowable Load} = (\text{Allowable Stress})(\text{Area})$
- Tension/Compression: $P_{allow} = \sigma_{allow} A$ [Area: Net Area]
- Direct Shear: $P_{allow} = \tau_{allow} A$ [Area: Cross-sectional Area]
- Bearing: $P_{allow} = \sigma_b A_b$ [Area: Projected Area]

Design For Axial Loads and Direct Shear

- Design– The determination of structural properties in order for the structure to support given loads and perform its intended function.

- Design of Areas for Members in Simple Tension or Compression:

$$\text{Required Area} = \frac{\text{Load To Be Transmitted}}{\text{Allowable Stress}}$$

Summary

- Principal Objective– to determine stresses, strains, and displacements.
- Normal Stress: $\sigma = \frac{P}{A}$ Normal Strain: $\varepsilon = \frac{\delta}{L}$
- Stress-Strain Diagram expresses the mechanical behavior of various materials.
- Hooke's Law: [Applicable only in the linearly elastic region] $\sigma = E\varepsilon$
- Poisson's Ratio: $\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon'}{\varepsilon}$
- Bearing Stress: $\sigma_b = \frac{F_b}{A_b}$ Shear Stress: $\tau_{ave} = \frac{V}{A}$

Summary (cont.)

- Hooke's Law in Shear: [shear strain (γ) measures distortion] $\tau = G\gamma$
- Shear Modulus of Elasticity (G): $G = \dfrac{E}{2(1+\nu)}$
- Factor of Safety: $\text{Factor of Safety } n = \dfrac{\text{Actual Strength}}{\text{Required Strength}}$
- Yield Strength is used for ductile materials; Ultimate Strength if brittle.
- Factor of Safety helps determine Allowable Stresses and Allowable Loads.
- Required areas of members can be determined from these allowable factors.

Free-body diagram

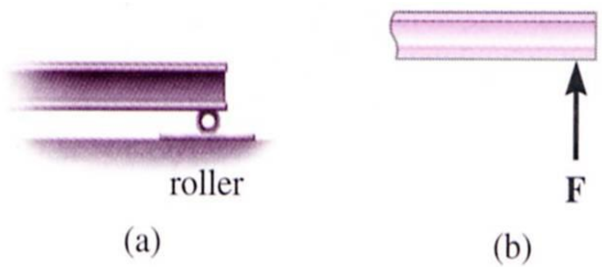


Fig. 4–2

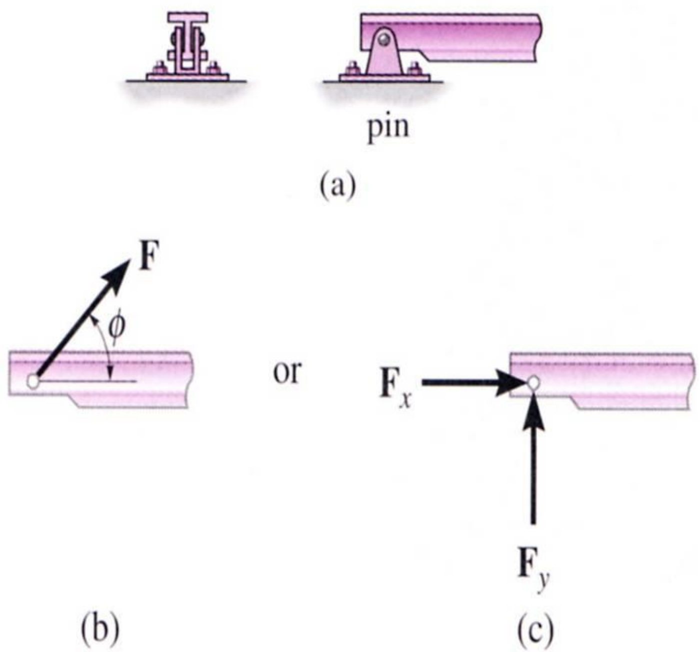


Fig. 4–3

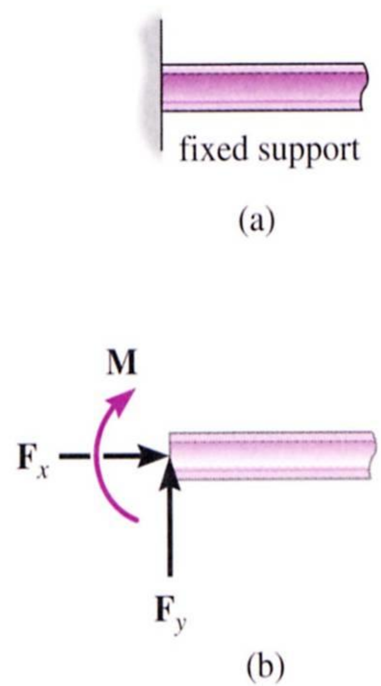
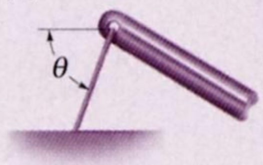
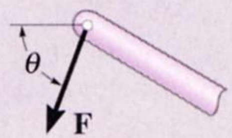
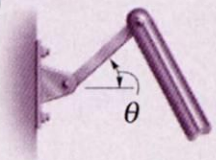
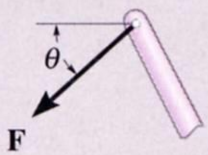
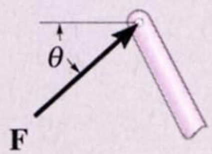
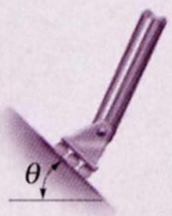
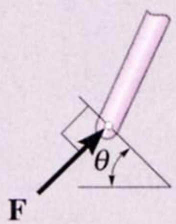


Fig. 4–4

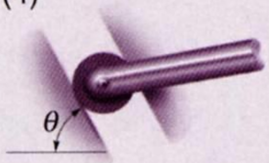
Free-body diagram

TABLE 4–1 • Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

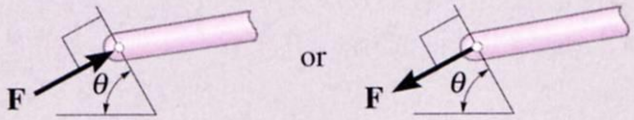
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

Free-body diagram

(4)

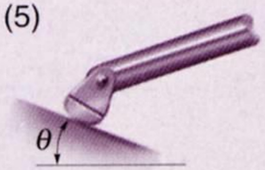


roller or pin in
confined smooth slot

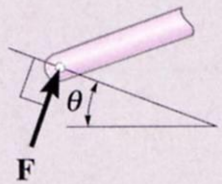


One unknown. The reaction is a force which acts perpendicular to the slot.

(5)

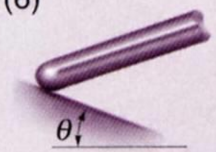


rocker

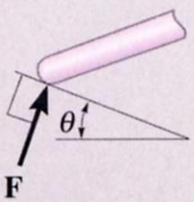


One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

(6)

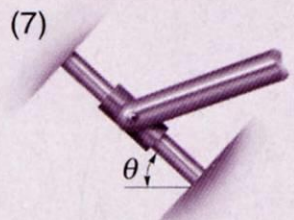


smooth contacting
surface

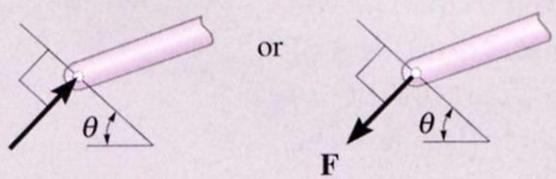


One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

(7)



member pin connected
to collar on smooth rod

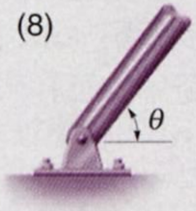
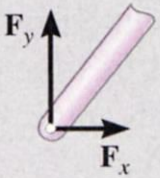
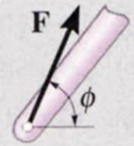
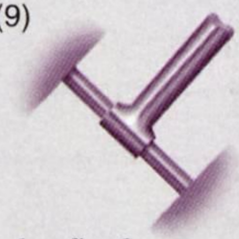
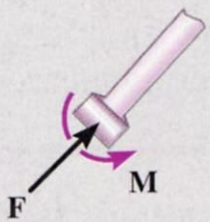
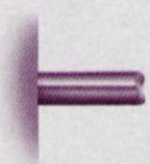
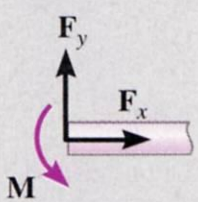
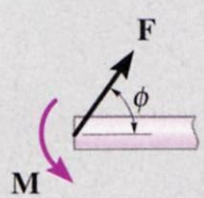


One unknown. The reaction is a force which acts perpendicular to the rod.

continued

Free-body diagram

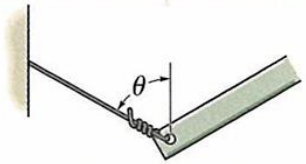
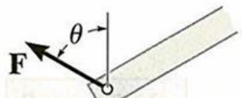
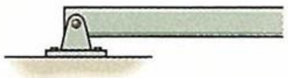
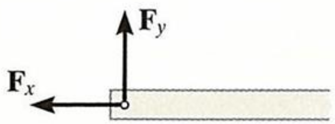
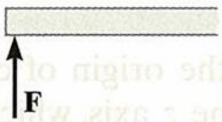
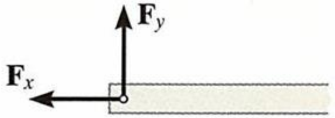
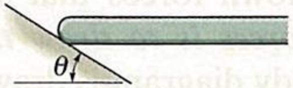
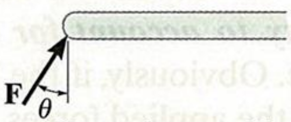
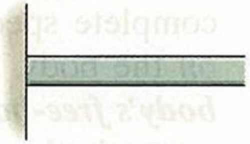
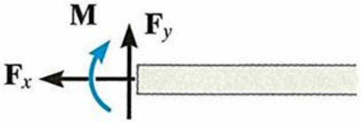
TABLE 4–1 • *Continued*

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge	 or 	Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support	 or 	Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

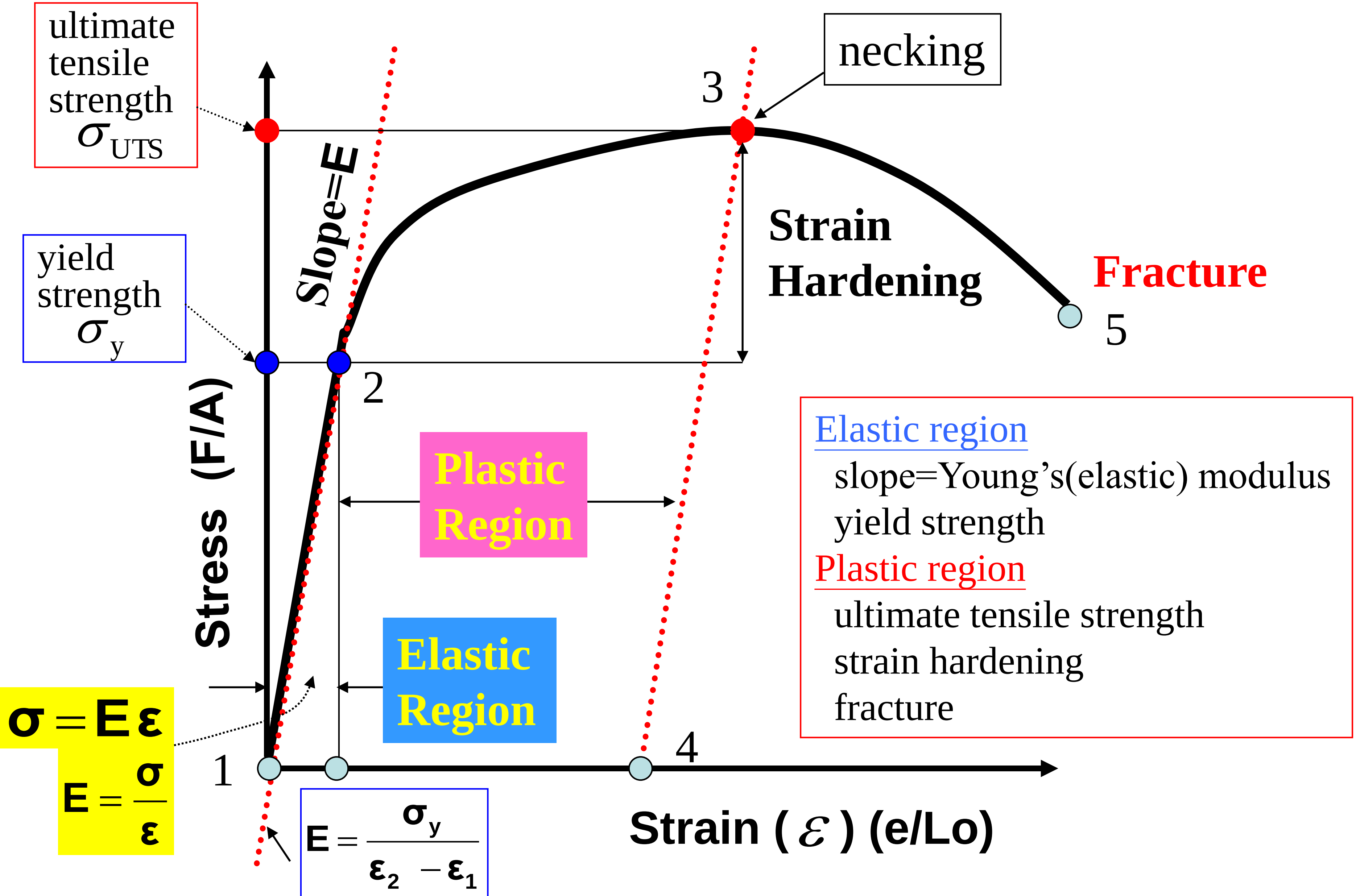
Support reactions

Support Reactions.

TABLE 1–1

Type of connection	Reaction	Type of connection	Reaction
 Cable	 One unknown: F	 External pin	 Two unknowns: F_x, F_y
 Roller	 One unknown: F	 Internal pin	 Two unknowns: F_x, F_y
 Smooth support	 One unknown: F	 Fixed support	 Three unknowns: F_x, F_y, M

Conventional Stress-Strain Diagram



Stress-Strain Diagram: Elastic region

- **Elastic Region (Point 1 –2)**

  - The material will return to its original shape after the material is unloaded(like a rubber band).

  - The stress is linearly proportional to the strain in this region.

$$\boxed{\sigma = E\varepsilon} \quad \text{or} \quad \boxed{E = \frac{\sigma}{\varepsilon}}$$

σ **: Stress(Mpa)**

E **: *Elastic modulus* (*Young's Modulus*) (GPa)**

ε **: Strain (mm/mm)**

- **Point 2 : Yield Strength : a point at which permanent deformation occurs. (If it is passed, the material will no longer return to its original length.)**

Stress-Strain Diagram: Plastic region

- **Plastic Region (Point 2 –3)**

  - If the material is loaded beyond the yield strength, the material will not return to its original shape after unloading.

  - It will have some permanent deformation.

  - If the material is unloaded at Point 3, the curve will proceed from Point 3 to Point 4. The slope will be the as the slope between Point 1 and 2.

  - The distance between Point 1 and 4 indicates the amount of permanent deformation.

Stress-Strain Diagram: Plastic region

- **Strain Hardening**

  - If the material is loaded again from Point 4, the curve will follow back to Point 3 with the same *Elastic Modulus(slope)*.

  - The material now has a higher yield strength of Point 4.

  - Raising the yield strength by permanently straining the material is called *Strain Hardening*.

Stress-Strain Diagram: Plastic region

- **Tensile Strength (Point 3)**

  - The largest value of stress on the diagram is called *Tensile Strength(TS)* or *Ultimate Tensile Strength (UTS)*

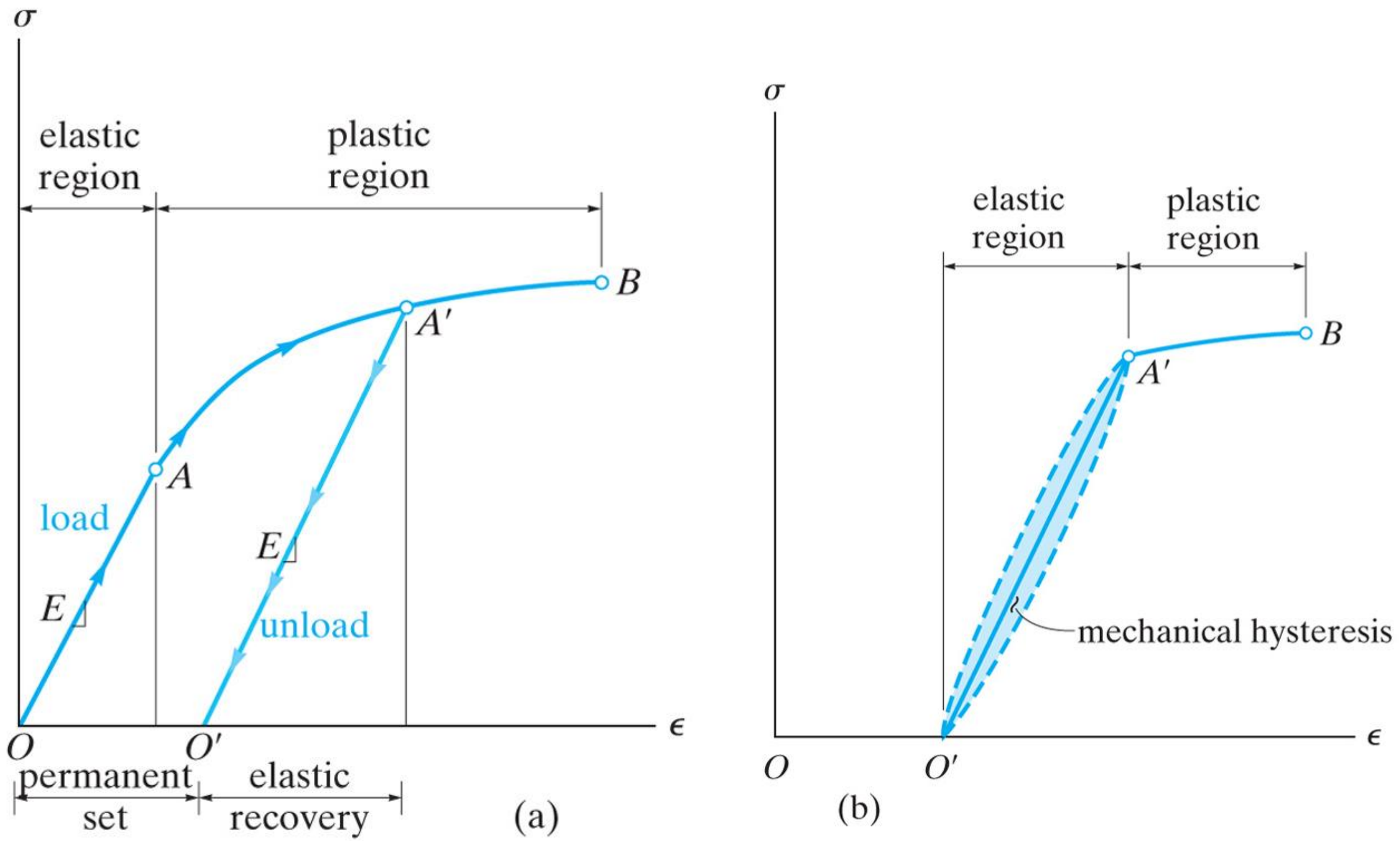
  - It is the maximum stress which the material can support without breaking.

- **Fracture (Point 5)**

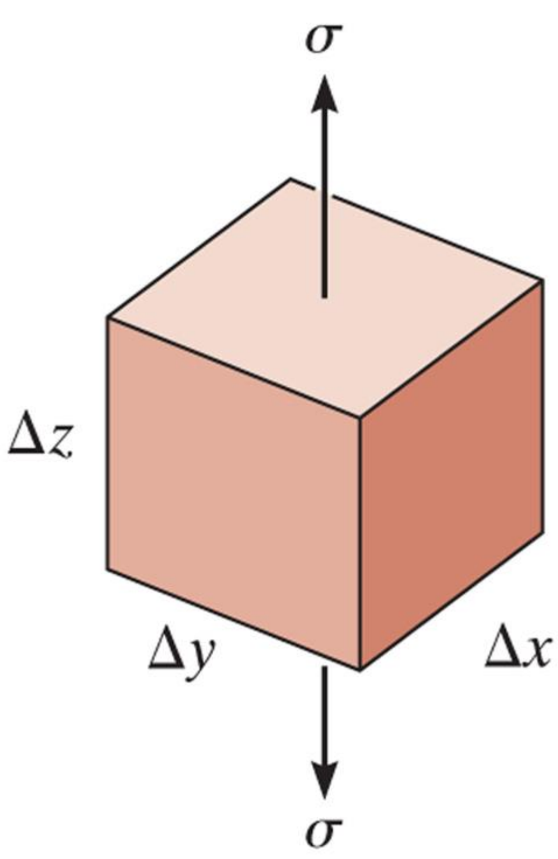
  - If the material is stretched beyond Point 3, the stress decreases as necking and non-uniform deformation occur.

  - Fracture will finally occur at Point 5.

Strain hardening



Strain energy



$$\Delta F = \sigma \Delta A = \sigma(\Delta x \Delta y)$$

$$\Delta U = \frac{1}{2}\Delta F(\varepsilon \Delta z) = \frac{1}{2}\sigma\varepsilon(\Delta x \Delta y \Delta z)$$

$$\Delta U = \frac{1}{2}\sigma\varepsilon\Delta V$$ Strain-energy

$$u = \frac{\Delta U}{\Delta V} = \frac{1}{2}\sigma\varepsilon$$ Strain-energy density

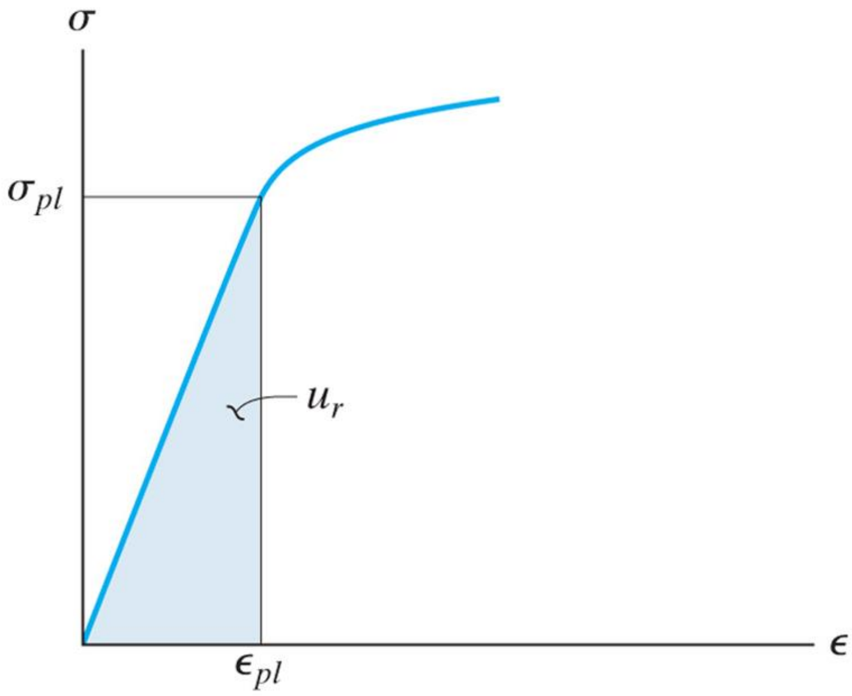
Modulus of resilience

Strain-energy density

$$u = \frac{1}{2}\frac{\sigma^2}{E}$$

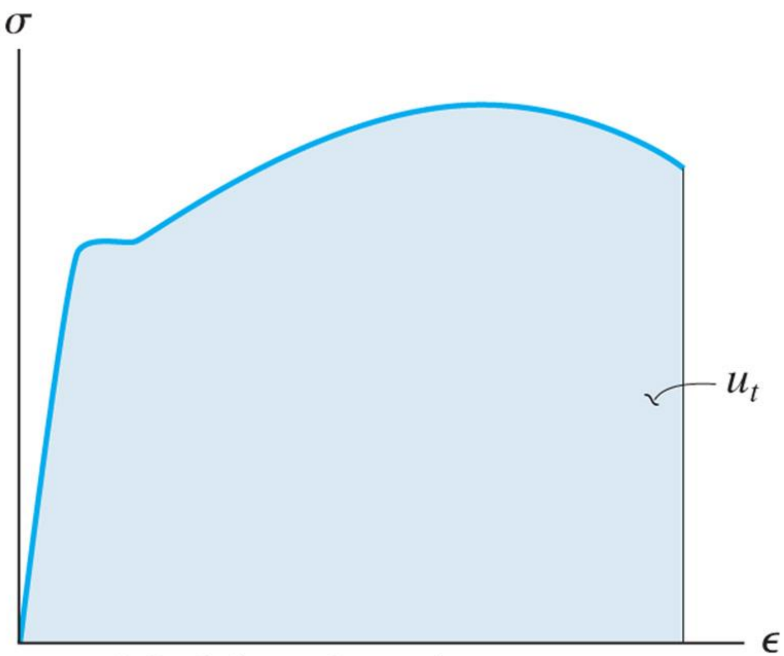
Modulus of resilience

$$u = \frac{1}{2}\sigma_{pl}\varepsilon_{pl} = \frac{1}{2}\frac{\sigma_{pl}^{\ 2}}{E}$$

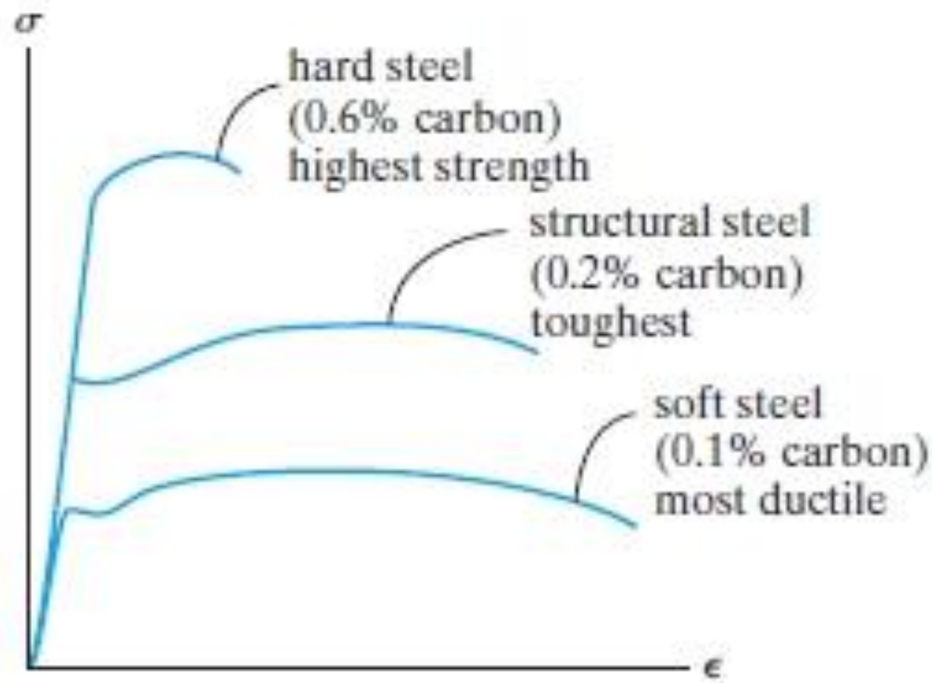


Modulus of resilience u_r

Modulus of toughness



Modulus of toughness u_t



Material Properties

1) Strength

- <u>**Measure of the material property to resist deformation and to maintain its shape**</u>
- **It is quantified in terms of yield stress or ultimate tensile strength.**
- **High carbon steels and metal alloys have higher strength than pure metals.**
- **Ceramic also exhibit high strength characteristics.**

Material Properties

2) Ductility

- **Measure of the material property to deform before failure.**
- **It is quantified by reading the value of strain at the fracture point on the stress strain curve.**
- **Example of ductile material :**

  low carbon steel

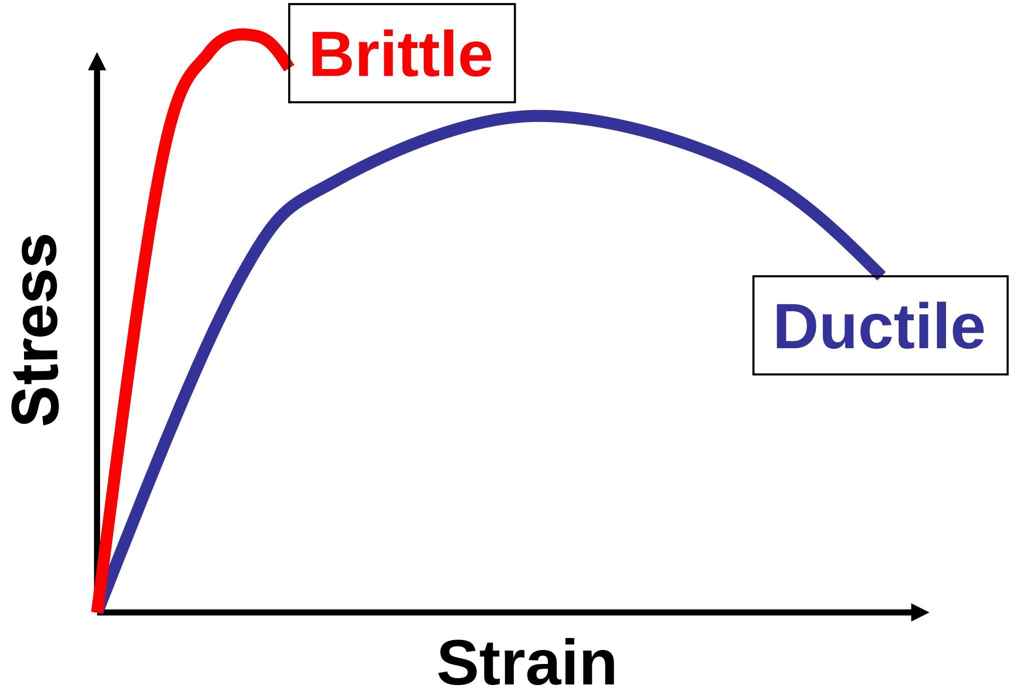
  aluminum

  bubble gum

Material Properties

3) Brittleness

- **<u>Measure of the material's *inability* to deform before failure.</u>**
- **The <u>opposite of ductility</u>.**
- **Example of ductile material : glass, high carbon steel, ceramics**



Material Properties

4) Toughness

- **Measure of the material ability to absorb *energy*.**

- **It is measured by two methods.**

a) Integration of stress strain curve

- **Slow absorption of energy**

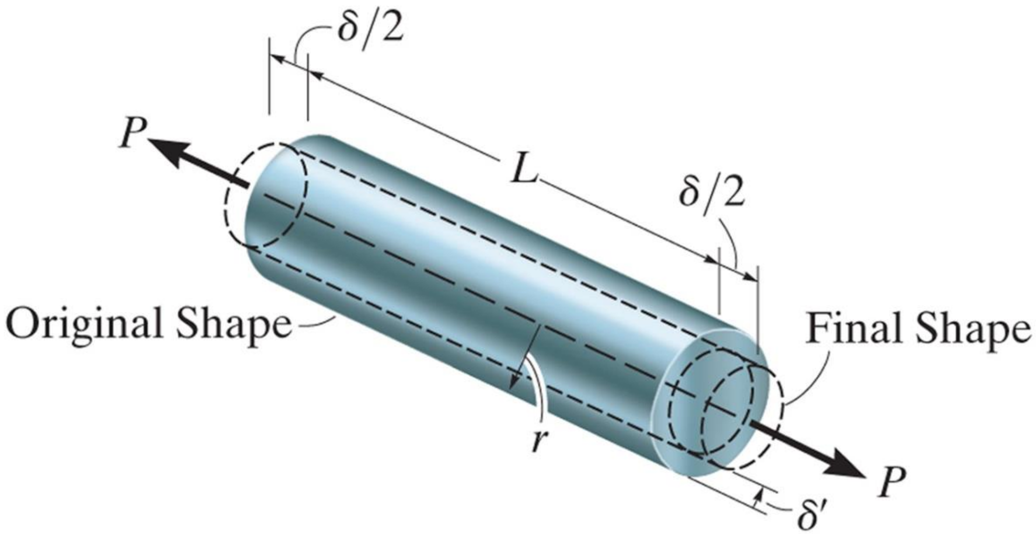
- **Absorbed energy per unit volume**

unit : $(lb/in^2) * (in/in) = lb \cdot in/in^3$

b) Charpy test

- ***Impact toughness* can be measured.**

Poisson's ratio



$$\varepsilon_{long} = \frac{\delta}{L}$$

$$\varepsilon_{lat} = \frac{\delta'}{L}$$

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$$